Maths Calculation Guidance

This guidance has been largely adapted from the White Rose Maths Hub Calculation Policy with further material added. It is a working document and will be revised and amended as necessary. Many variations have been included to provide teachers with a range of tools to support pupils in their grasp of number and calculation. To ensure consistency for pupils, it is important that that the mathematical language used in Maths lessons reflects the vocabulary used throughout this policy.

Date	Spring 2024
Next Review Due	Spring 2027
Headteacher	Z Driver

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Early Maths

> Young learners' future understanding of mathematics requires an early foundation based on a high-quality, challenging and accessible mathematics education… Early childhood educators should actively introduce mathematical concepts, methods, and language through a variety of appropriate experiences. Teachers should guide children in seeing connections of ideas within mathematics and across the curriculum. They must encourage children to communicate, explaining their thinking as they interact with important mathematics in deep and sustained ways. (NCETM, 2013)

At Burbage Junior School we believe that these principles are fundamental for pupils of all ages and will work closely with our colleagues at the Infant school to ensure that we build on these solid foundations. The Infants school exclusively use the White Rose calculation policy, however, we have chosen to largely follow it with a few alterations.

Recommended practice delivering a mastery approach

True mastery aims to develop all children's mathematical understanding at the same pace. As far as possible, children should be accessing the same learning. Differentiation should primarily be through support, scaffolding and deepening, not through task.

Consistency in language is essential for pupils to understand the concepts presented in mathematics. If other, 'child-friendly' terminology is used, this must be alongside the current terminology recommended by Maths Specialists. Use of sentence stems tie in with our school's spoken language curriculum as well as allowing pupils to express themselves appropriately. Stem sentences are used in order to support children's learning and aid working memory.

Concrete, pictorial, abstract (CPA) concepts should not be confused as differentiation for lower, middle, higher attaining children. CPA is an approach to be used with the whole class and teachers should promote each area as equally valid. Manipulatives in particular must not be presented as a resource to support the less confident or lower attaining pupils.

> Children aged seven to ten years old work in primarily concrete ways and that the abstract notions of mathematics may only be accessible to them through embodiment in practical resources. (Piaget, 1951)

> Used well, manipulatives can enable pupils to inquire themselves- becoming independent learners and thinkers. They can also provide a common language with which to communicate cognitive models for abstract ideas. (Drury, 2015)

> Real things and structured images enable children to understand the abstract. The concrete and the images are a means for children to understand the symbolic so it's important to move between all modes to allow children to make connections. (Morgan, 2016)

The abstract should run alongside the concrete and pictorial stage as this enables pupils to better understand mathematical statements and concepts.

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Addition

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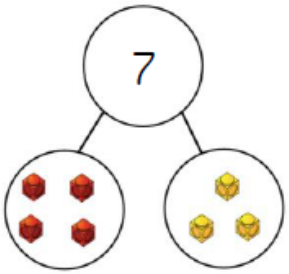
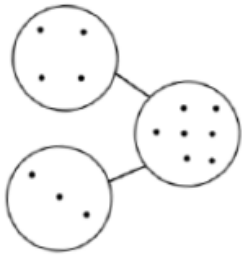
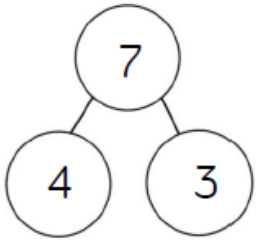
Part-Whole Model





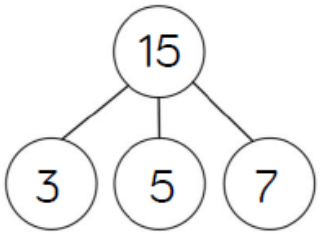


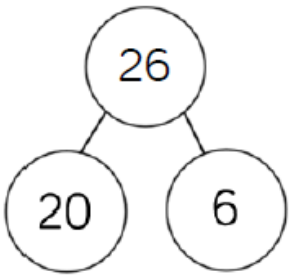
$7 = 4 + 3$

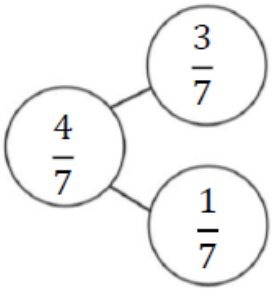
$7 = 3 + 4$

$7 - 3 = 4$

$7 - 4 = 3$







Benefits

This part-whole model supports children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model.

When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part.

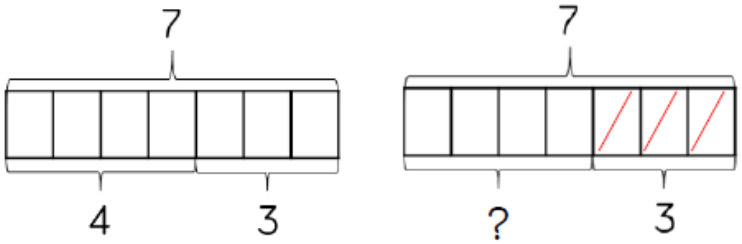
Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.

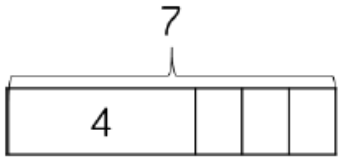
Bar Model (single)

Concrete

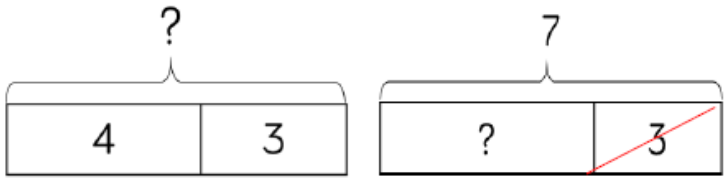
Discrete

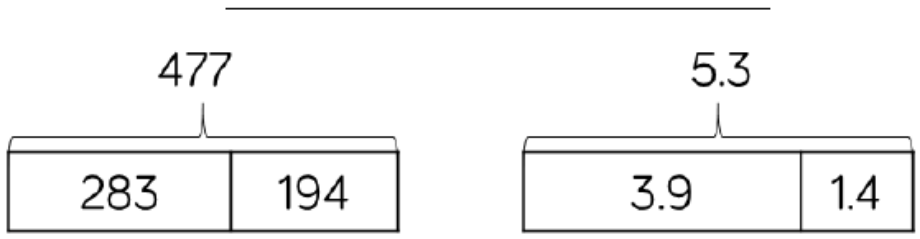


Combination



Continuous





Benefits

The single bar model is another type of a part-whole model that can support children in representing calculations to help them unpick the structure.

Cubes and counters can be used in a line as a concrete representation of the bar model.

Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.

The combination bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model.

Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.

In KS2, children can use bar models to represent larger numbers, decimals and fractions.

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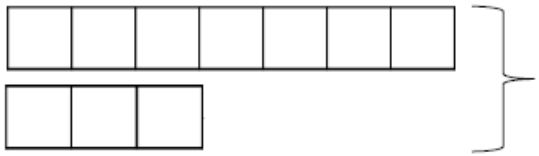
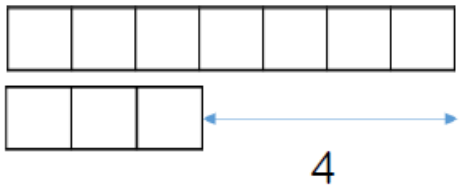
Bar Model (multiple)

Discrete

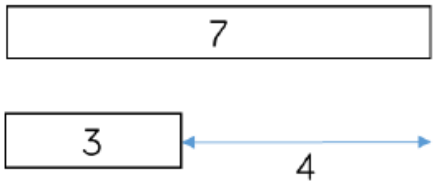


$7 + 3 = 10$

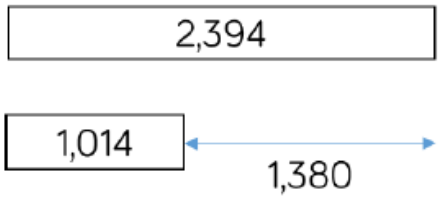


$7 - 3 = 4$

Continuous



$7 - 3 = 4$



$2{,}394 - 1{,}014 = 1{,}380$

Benefits

The multiple bar model is a good way to compare quantities whilst still unpicking the structure.

Two or more bars can be drawn, with a bracket labelling the whole positioned on the right hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers.

Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.

When working with smaller numbers, children can use cubes and a discrete model to find the difference. This supports children to see how counting on can help when finding the difference.

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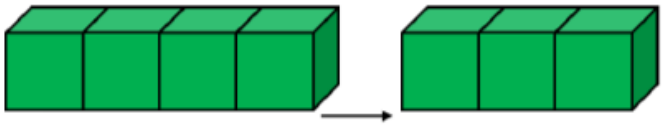
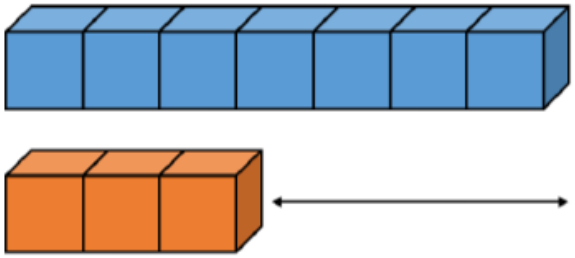
Cubes

$7 = 4 + 3$

$7 = 3 + 4$



$7 - 3 = 4$



$7 - 3 = 4$

Benefits

Cubes can be useful to support children with the addition and subtraction of one-digit numbers.

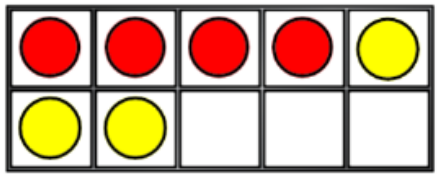
When adding numbers, children can see how the parts come together to make a whole. Children could use two different colours of cubes to represent the numbers before putting them together to create the whole.

When subtracting numbers, children can start with the whole and then remove the number of cubes that they are subtracting in order to find the answer. This model of subtraction is reduction, or take away.

Cubes can also be useful to look at subtraction as difference. Here, both numbers are made and then lined up to find the difference between the numbers.

Cubes are useful when working with smaller numbers but are less efficient with larger numbers as they are difficult to subitise and children may miscount them.

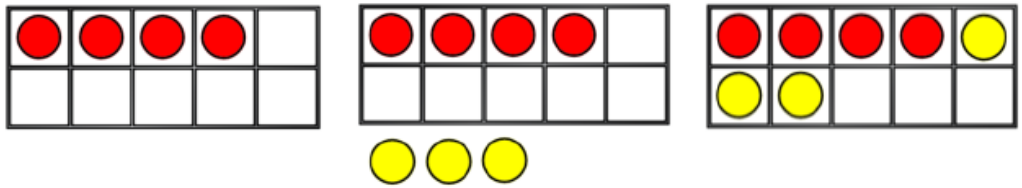
Ten Frames (within 10)



$4 + 3 = 7$
$3 + 4 = 7$
$7 - 3 = 4$
$7 - 4 = 3$

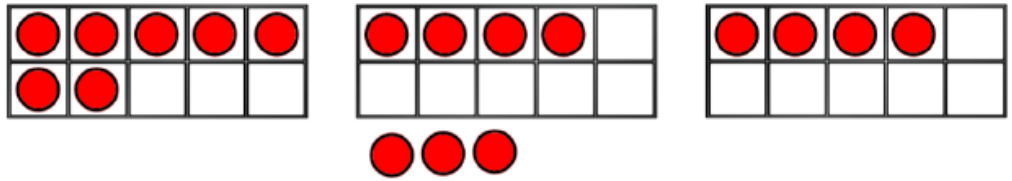
4 is a part.
3 is a part.
7 is the whole.

First | Then | Now



$4 + 3 = 7$

First | Then | Now



$7 - 3 = 4$

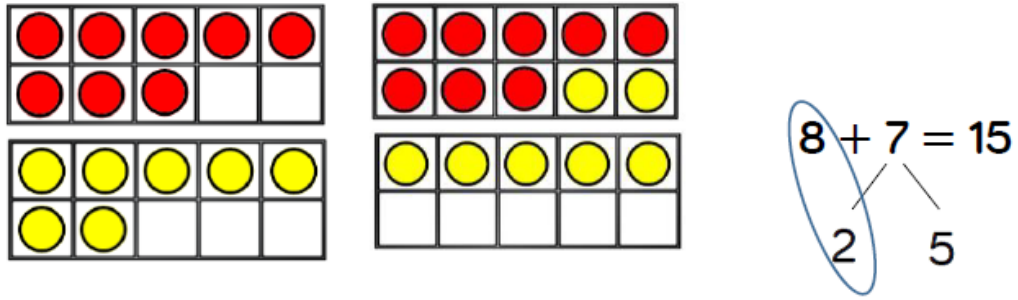
Benefits

When adding and subtracting within 10, the ten frame can support children to understand the different structures of addition and subtraction.

Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning.
Aggregation is a form of addition where parts are combined together to make a whole. Partitioning is a form of subtraction where the whole is split into parts. Using these structures, the ten frame can enable children to find all the number bonds for a number.

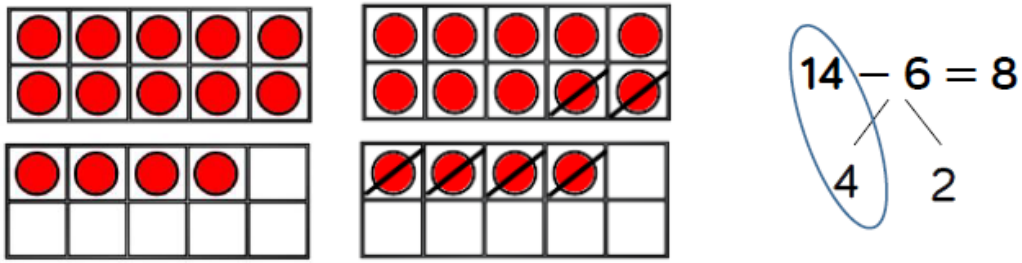
Children can also use ten frames to look at augmentation (increasing a number) and take-away (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help children understand the change e.g. First, there were 7 cars. Then, 3 cars left. Now, there are 4 cars.

Ten Frames (within 20)



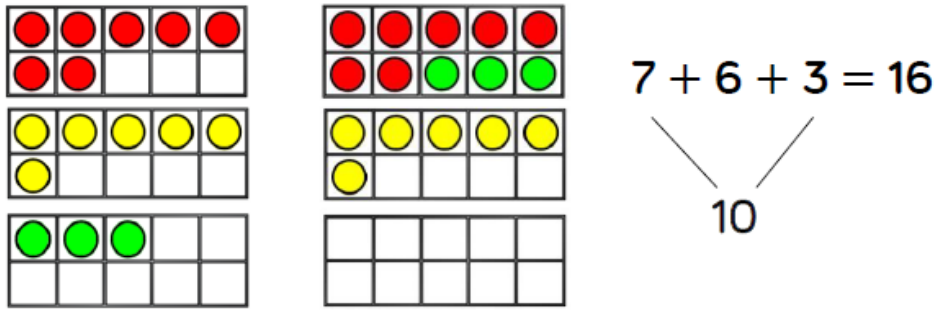
$8 + 7 = 15$

2 5



$14 - 6 = 8$

4 2



$7 + 6 + 3 = 16$

10

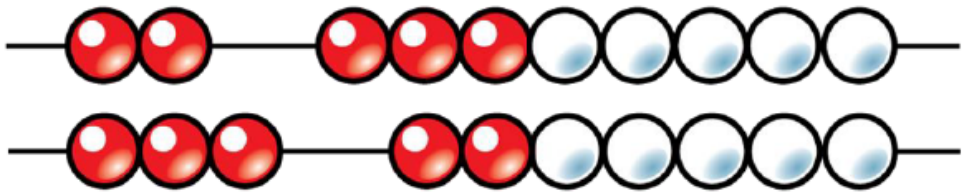
Benefits

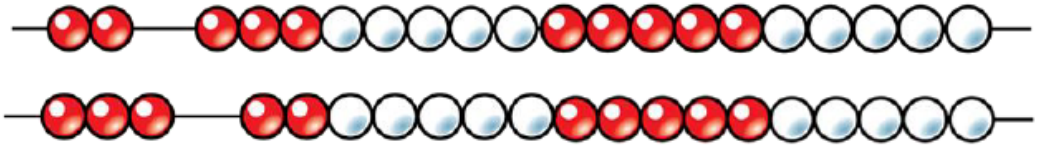
When adding two single digits, children can make each number on separate ten frames before moving part of one number to make 10 on one of the ten frames. This supports children to see how they have partitioned one of the numbers to make 10, and makes links to effective mental methods of addition.

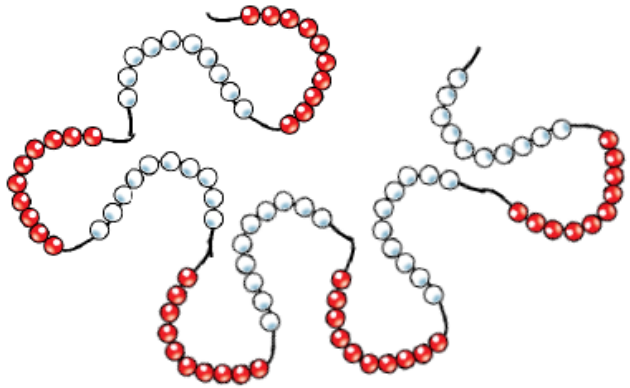
When subtracting a one-digit number from a two-digit number, firstly make the larger number on 2 ten frames. Remove the smaller number, thinking carefully about how you have partitioned the number to make 10, this supports mental methods of subtraction.

When adding three single-digit numbers, children can make each number on 3 separate 10 frames before considering which order to add the numbers in. They may be able to find a number bond to 10 which makes the calculation easier. Once again, the ten frames support the link to effective mental methods of addition as well as the importance of commutativity.

Bead Strings







Benefits

Different sizes of bead strings can support children at different stages of addition and subtraction.

Bead strings to 10 are very effective at helping children to investigate number bonds up to 10.
They can help children to systematically find all the number bonds to 10 by moving one bead at a time to see the different numbers they have partitioned the 10 beads into e.g. 2 + 8 = 10, move one bead, 3 + 7 = 10.

Bead strings to 20 work in a similar way but they also group the beads in fives. Children can apply their knowledge of number bonds to 10 and see the links to number bonds to 20.

Bead strings to 100 are grouped in tens and can support children in number bonds to 100 as well as helping when adding by making ten. Bead strings can show a link to adding to the next 10 on number lines which supports a mental method of addition.

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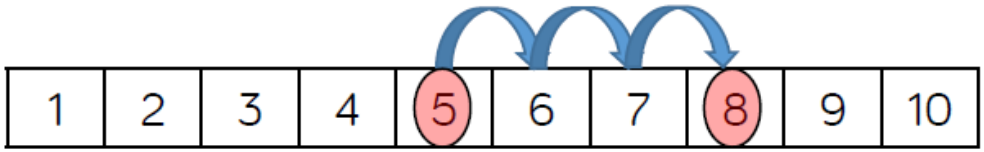
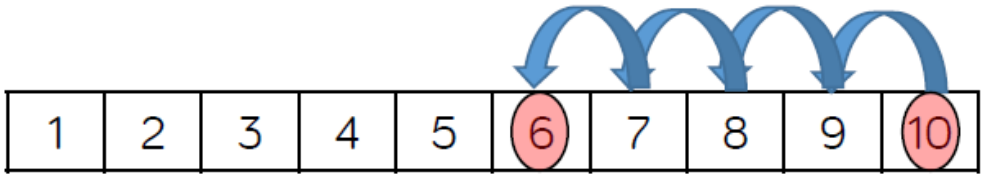
Number Tracks

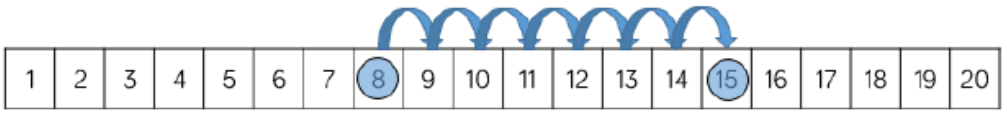
$5 + 3 = 8$



$10 - 4 = 6$



$8 + 7 = 15$



Benefits

Number tracks are useful to support children in their understanding of augmentation and reduction.

When adding, children count on to find the total of the numbers. On a number track, children can place a counter on the starting number and then count on to find the total.

When subtracting, children count back to find their answer. They start at the minuend and then take away the subtrahend to find the difference between the numbers.

Number tracks can work well alongside ten frames and bead strings which can also model counting on or counting back.

Playing board games can help children to become familiar with the idea of counting on using a number track before they move on to number lines.

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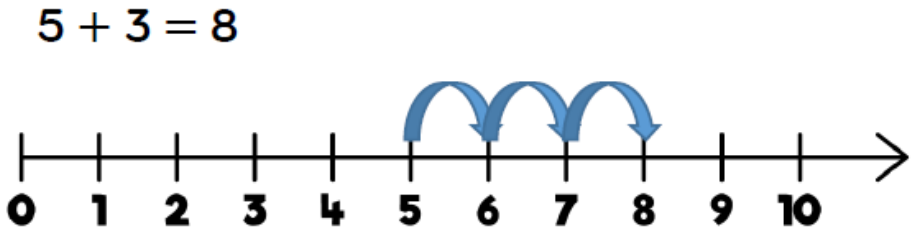
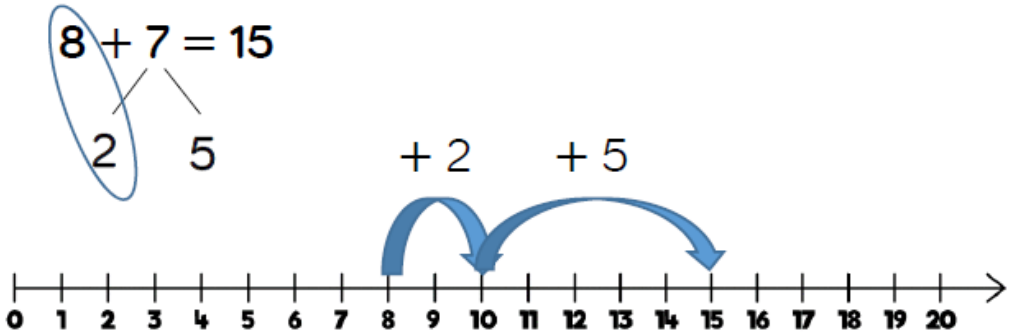
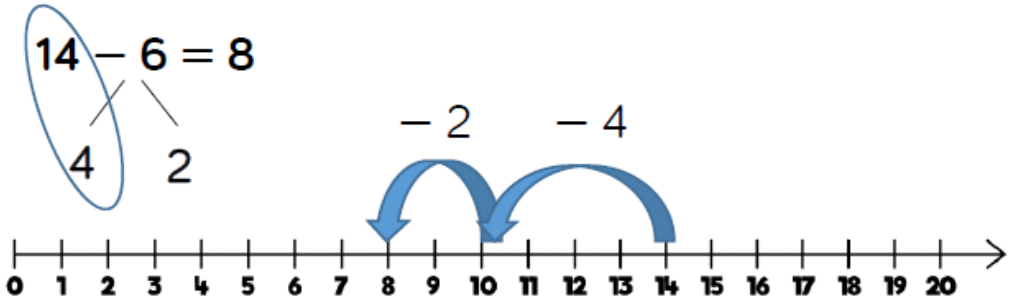
Number Lines (labelled)







Benefits

Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction.

Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the number track.

Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.

Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by ten frames so children can see how they partition the smaller number into the two separate jumps.

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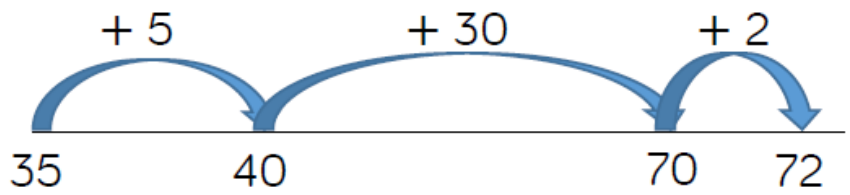
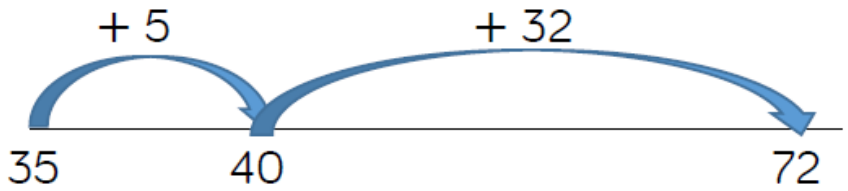
Number Lines (blank)

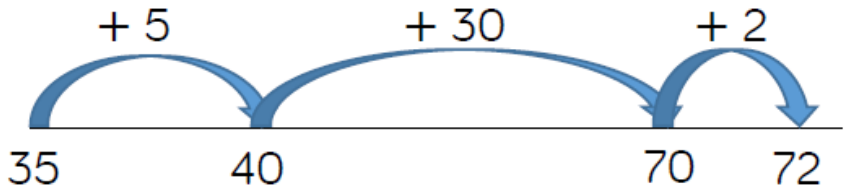
35 + 37 = 72



35 + 37 = 72



72 – 35 = 37



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Benefits

Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

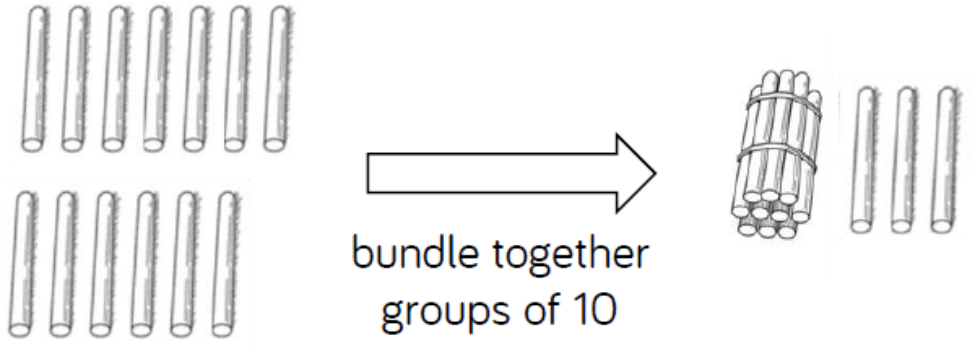
Developing from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately.

Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number.

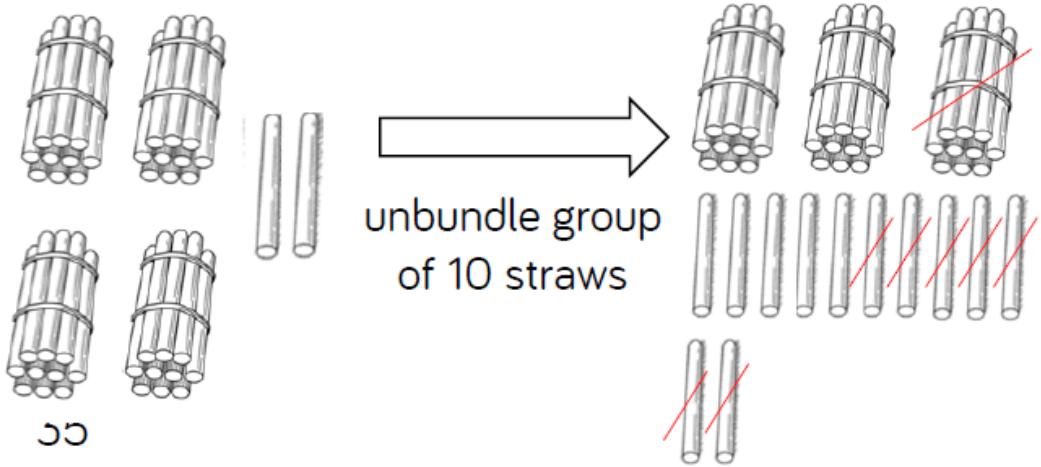
Blank number lines can also be used effectively to help children subtract by finding the difference between numbers. This can be done by starting with the smaller number and then counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.

Straws

$7 + 6 = 13$



$42 - 17 = 25$



Benefits

Straws are an effective way to support children in their understanding of exchange when adding and subtracting 2-digit numbers.

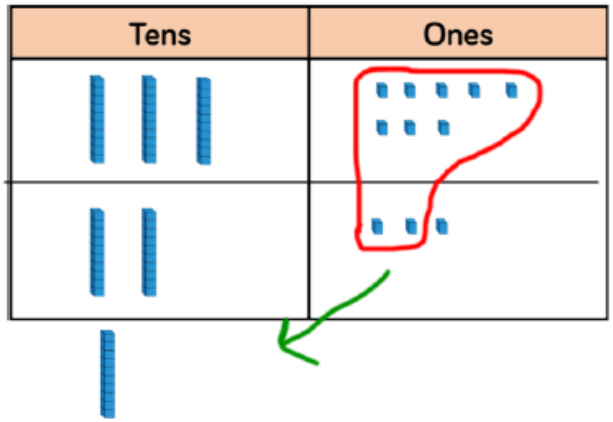
Children can be introduced to the idea of bundling groups of ten when adding smaller numbers and when representing 2-digit numbers. Use elastic bands or other ties to make bundles of ten straws.

When adding numbers, children bundle a group of 10 straws to represent the exchange from 10 ones to 1 ten. They then add the individual straws (ones) and bundles of straws (tens) to find the total.

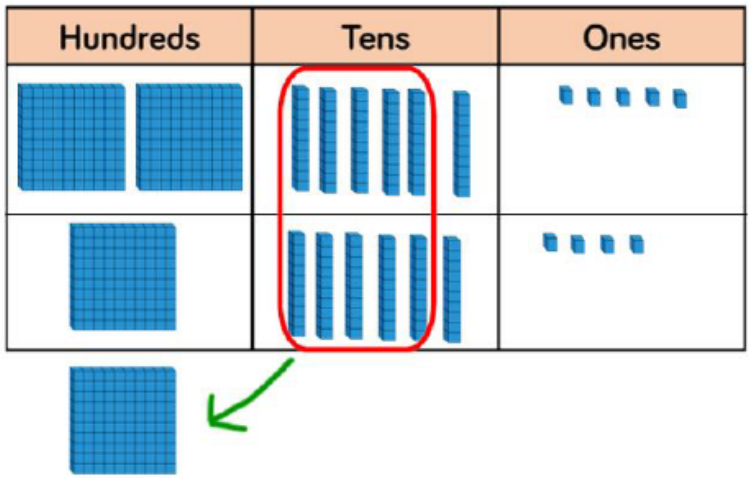
When subtracting numbers, children unbundle a group of 10 straws to represent the exchange from 1 ten to 10 ones.

Straws provide a good stepping stone to adding and subtracting with Base 10/Dienes.

Base 10/Dienes (addition)



$$\begin{array}{r} 38 \\ +\ 23 \\ \hline 61 \\ \hline 1 \end{array}$$



$$\begin{array}{r} 265 \\ +\ 164 \\ \hline 429 \\ \hline 1 \end{array}$$

Benefits

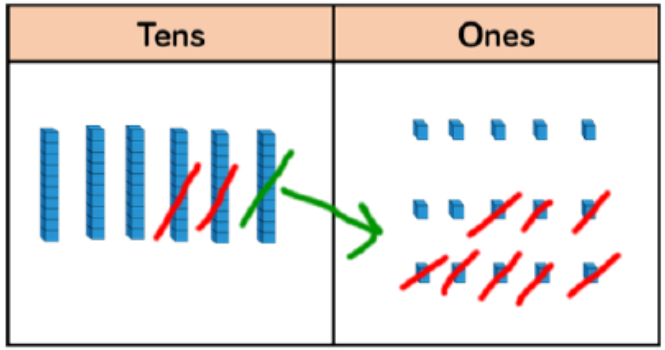
Using Base 10 or Dienes is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

Children should first add without an exchange before moving on to addition with exchange.. The representation becomes less efficient with larger numbers due to the size of Base 10. In this case, place value counters may be the better model to use.

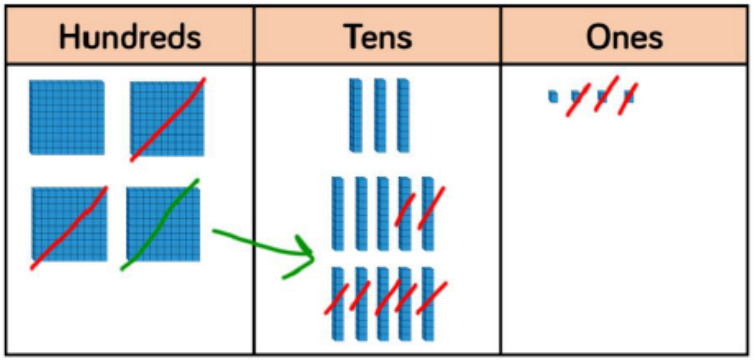
When adding, always start with the smallest place value column. Here are some questions to support children.
How many ones are there altogether?
Can we make an exchange? (Yes or No)
How many do we exchange? (10 ones for 1 ten, show exchanged 10 in tens column by writing 1 in column)
How many ones do we have left? (Write in ones column)
Repeat for each column.

``

Base 10/Dienes (subtraction)



$$\begin{array}{r} {}^{5}\not{6}\,{}^{1}\not{5} \\ -\ 28 \\ \hline 37 \end{array}$$



$$\begin{array}{r} {}^{3}\not{4}\,{}^{1}\not{3}5 \\ -\ 273 \\ \hline 162 \end{array}$$

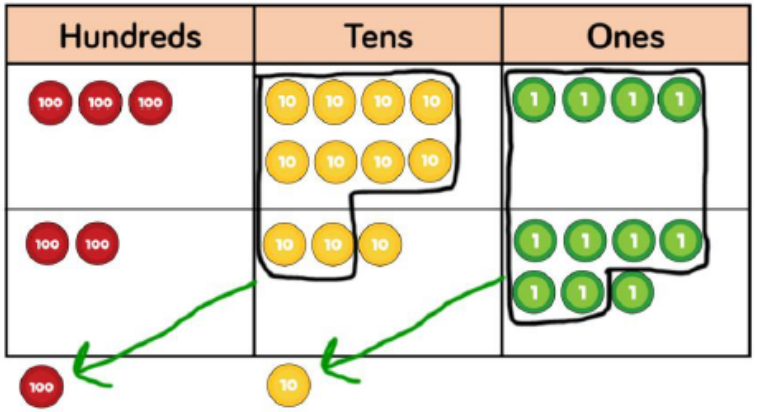
Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

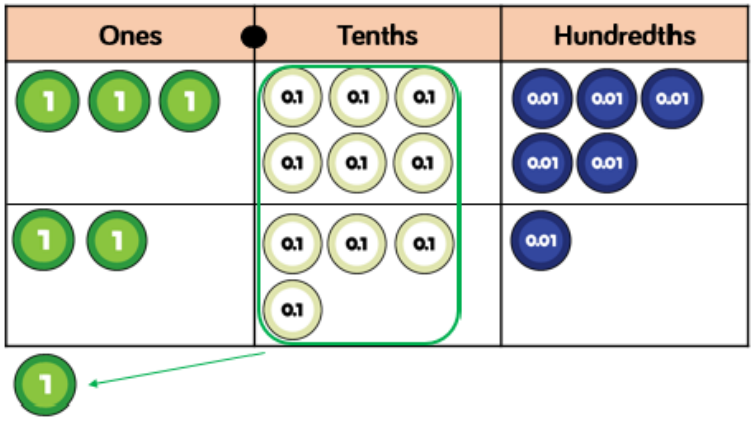
Children should first subtract without an exchange before moving on to subtraction with exchange. When building the model, children should just make the minuend using Base 10, they then subtract the subtrahend. Highlight this difference to addition to avoid errors by making both numbers. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

This model is efficient with up to 4-digit numbers. Place value counters are more efficient with larger numbers and decimals.

Place Value Counters (addition)



$$\begin{array}{r} 384 \\ +\ 237 \\ \hline 621 \\ \hline 1\ \ 1 \end{array}$$



$$\begin{array}{r} 3.65 \\ +\ 2.41 \\ \hline 6.06 \\ \hline 1 \end{array}$$

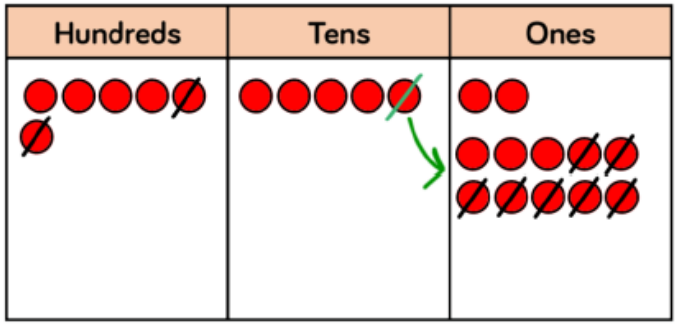
Benefits

Using place value counters is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

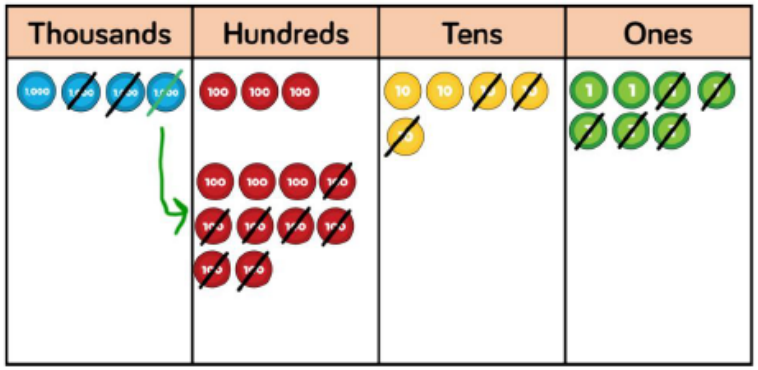
Children should first add without an exchange before moving on to addition with exchange. Different place value counters can be used to represent larger numbers or decimals. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When adding money, children can also use coins to support their understanding. It is important that children consider how the coins link to the written calculation especially when adding decimal amounts.

Place Value Counters (Subtraction)



$$\begin{array}{r} {}^{4}\cancel{6}{}^{1}\cancel{5}2 \\ -\ 207 \\ \hline 445 \end{array}$$



$$\begin{array}{r} {}^{3}\cancel{4}{}^{1}357 \\ -\ 2735 \\ \hline 1622 \end{array}$$

Benefits

Using place value counters is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When building the model, children should just make the minuend using counters, they then subtract the subtrahend. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

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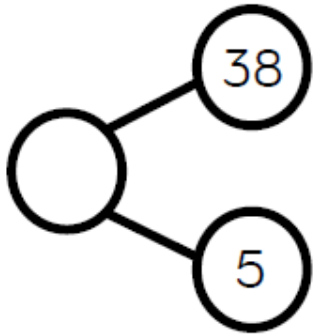
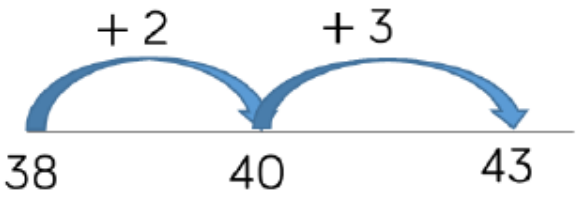
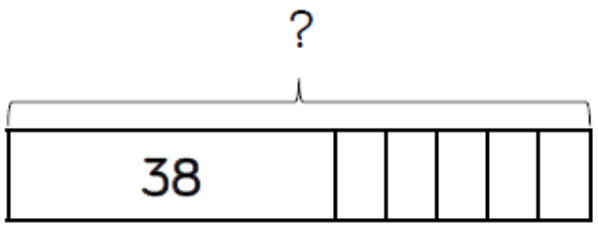
Skill: Add 1-digit and 2-digit numbers to 100

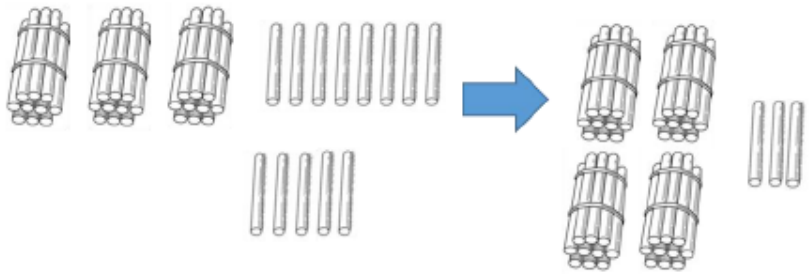
Year: 2/3







$$38 + 5 = 43$$

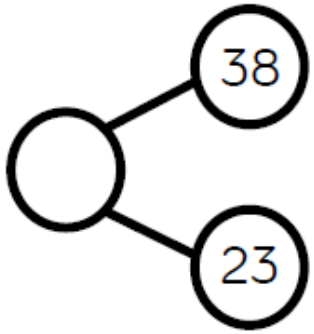
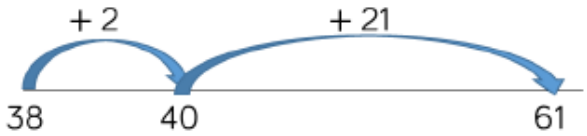
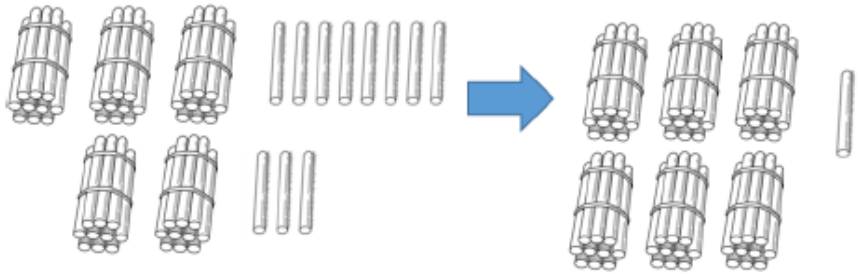
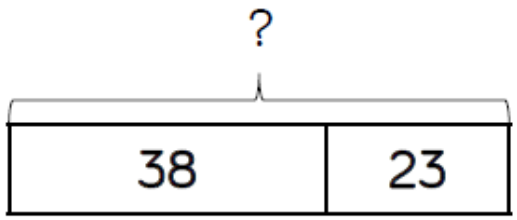
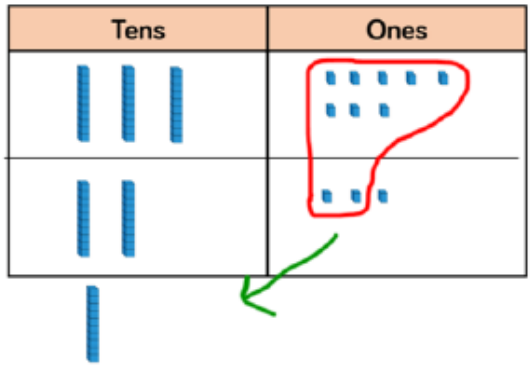
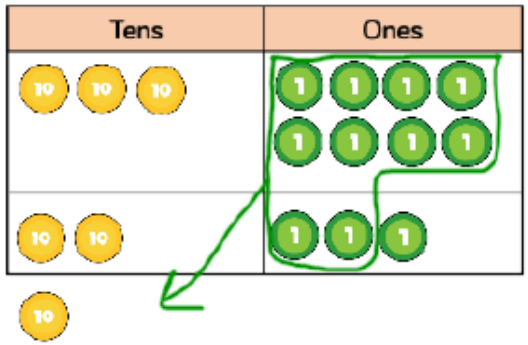


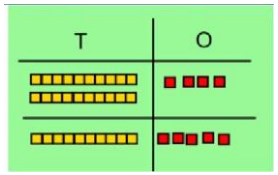
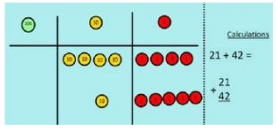
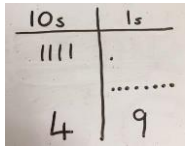
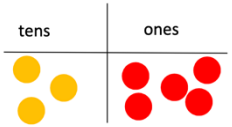
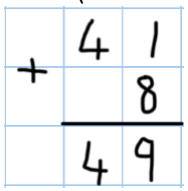
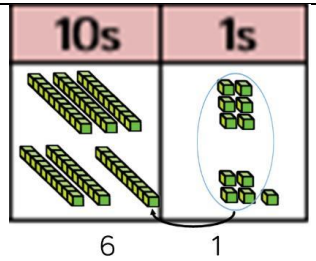
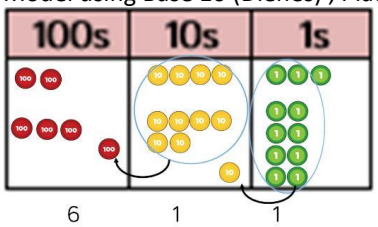
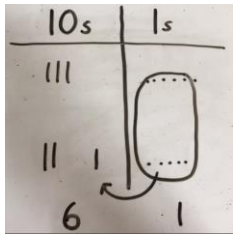
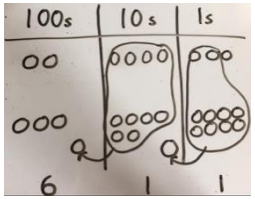
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

When adding single digits to a two-digit number, children should be encouraged to count on from the larger number.

They should also apply their knowledge of number bonds to add more efficiently e.g. 8 + 5 = 13 so 38 + 5 = 43.

Hundred squares and straws can support children to find the number bond to 10.

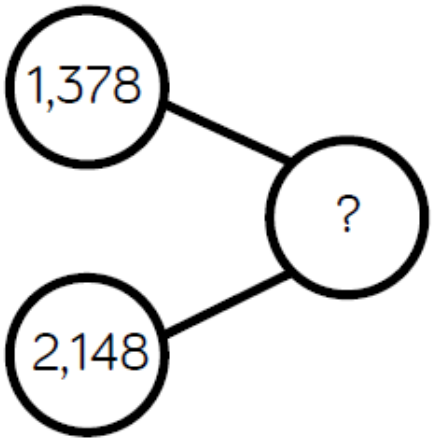
Skill: Add two 2-digit numbers to 100	Year: 2/3
    **38 + 23 = 61**  38 + 23 61 1 	Children can use a blank number line and other representations to count on to find the total. Encourage them to jump to multiples of 10 to become more efficient. From Year 3, encourage children to use the formal column method when calculating alongside straws, base 10 or place value counters. As numbers become larger, straws become less efficient.

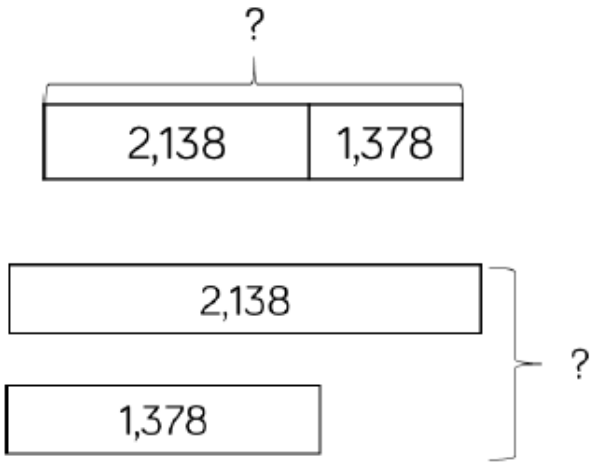
Year 3 Addition			
Objective/ Strategy	Concrete	Pictorial	Abstract
Column Addition— no regrouping (friendly numbers) Add two or three 2 or 3digit numbers.	 Use Base 10, Place Value Counters Add together the ones first, then the tens. 	Children move to drawing the counters using a tens and one frame.  	41 + 8 1 + 8= 9 40 + 0 (4 tens + 0 tens) = 40 (or 4 tens)  223 + 114 337 Add the ones first, then the tens, then the hundreds.
Column Addition with regrouping.	 I can exchange ten 1s for one 10. I can exchange ten 10s for one 100. I can exchange ten 100s for one 1000. Model using Base 10 (Dienes) , Place Value Counters. 	  Children can draw a representation of the grid to further support their understanding, **carrying both underneath (as shown in White Rose) and above.**	Formal column addition showing exchange. 243 +368 611 1 1 If children are struggling: 200 + 40 + 3 300 + 60 + 8 500 + 100 + 11 = 611 start by partitioning the numbers before formal column to show the exchange.

``

Skill: Add numbers with up to 4 digits

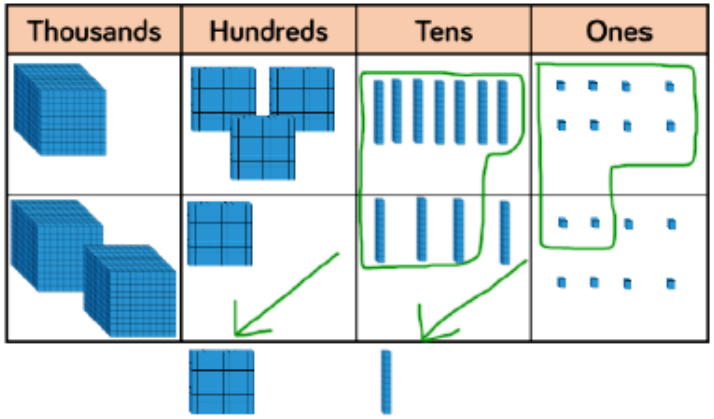
Year: 4

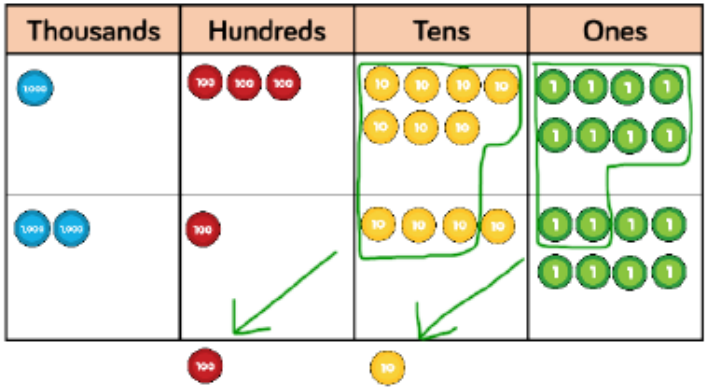




	1	3	7	8
+	2	1	4	8
	3	5	2	6
		1	1	

1,378 + 2,148 = 3,526





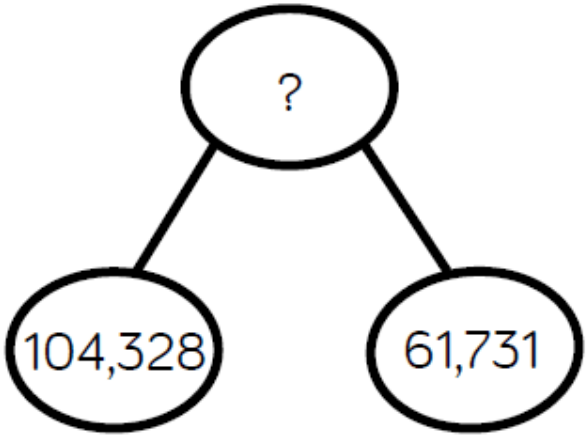
Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 4 digits.

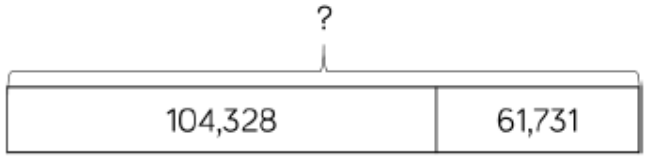
Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

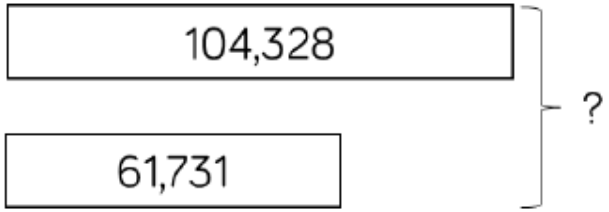
Plain counters on a place value grid can also be used to support learning.

Skill: Add numbers with more than 4 digits

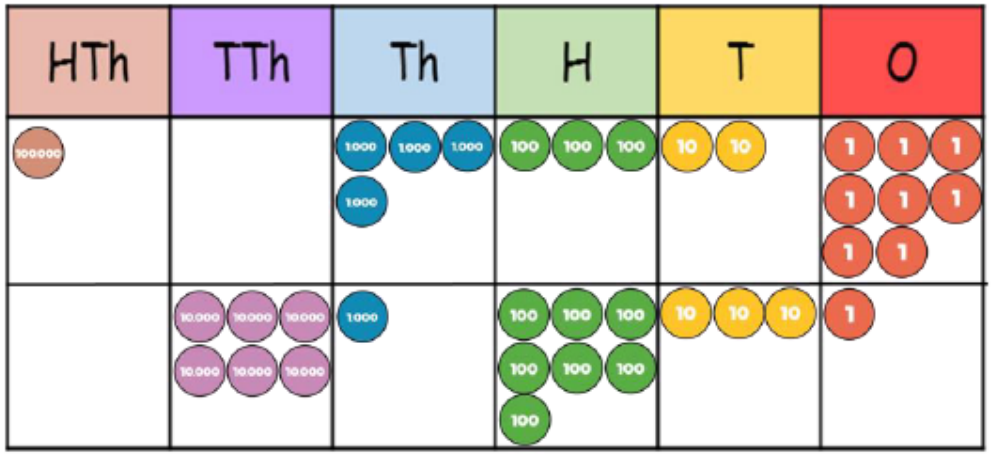
Year: 5/6







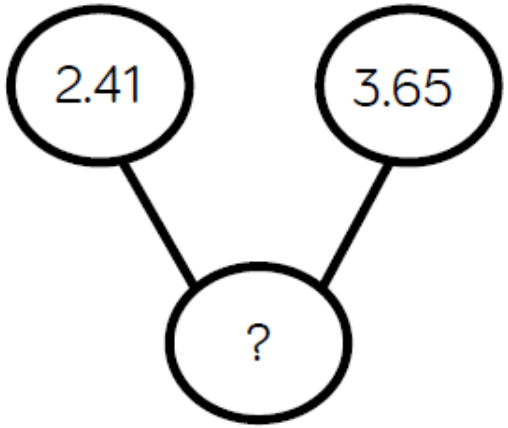
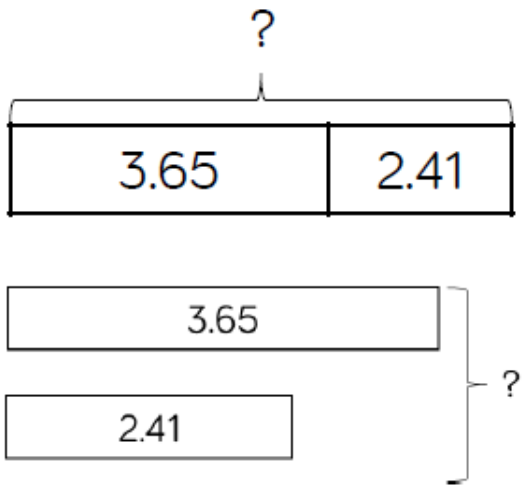
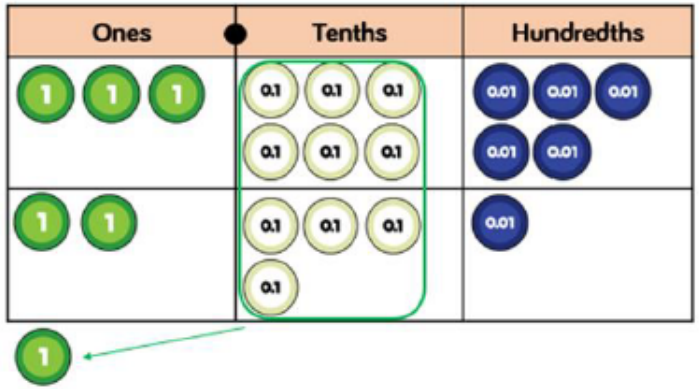
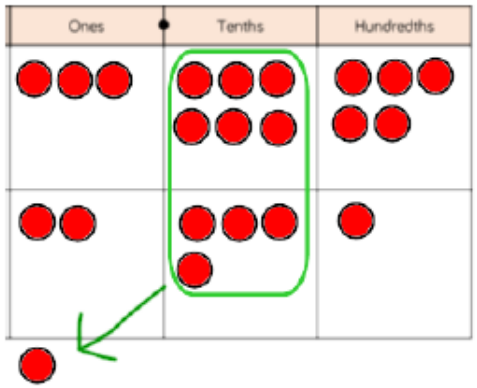
104,328 + 61,731 = 166,059



1	0	4	3	2	8
+	6	1	7	3	1
1	6	6	0	5	9
		1			

Place value counters or plain counters on a place value grid are the most effective concrete resources when adding numbers with more than 4 digits.

At this stage, children should be encouraged to work in the abstract, using the column method to add larger numbers efficiently.

Skill: Add with up to 3 decimal places	Year: 5
  $$\begin{array}{r} 3.65 \\ +\ 2.41 \\ \hline 6.06 \\ \hline 1 \end{array}$$ **$3.65 + 2.41 = 6.06$**  	Place value counters and plain counters on a place value grid are the most effective manipulatives when adding decimals with 1, 2 and then 3 decimal places. Ensure children have experience of adding decimals with a variety of decimal places. This includes putting this into context when adding money and other measures.

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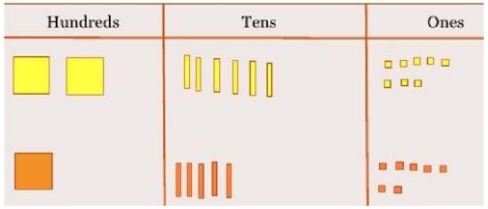
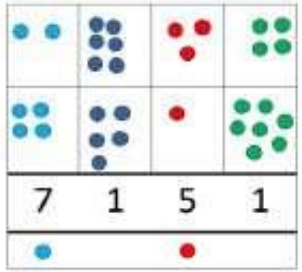
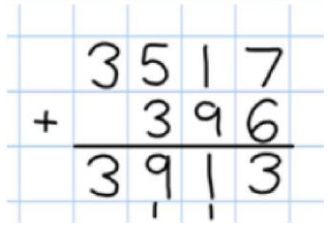
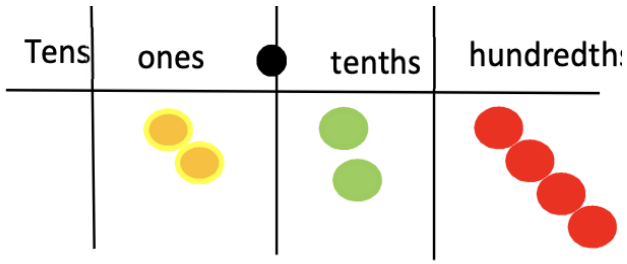
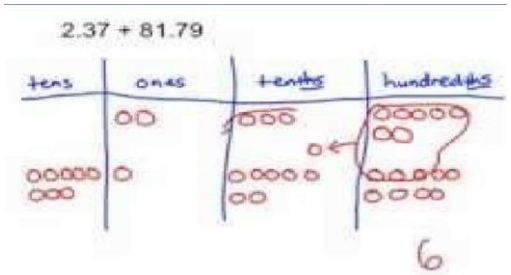
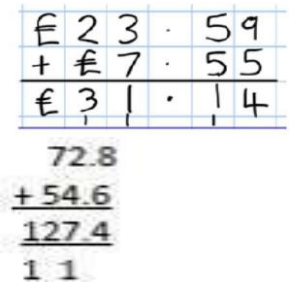
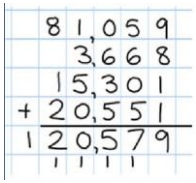
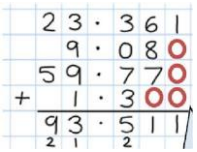
Year 4 – 6 Addition			
Objective/ Strategy	Concrete	Pictorial	Abstract
Y4—add numbers with up to 4 digits	Children continue to use base 10 or place value counters to add, exchanging as in Year 3. I can exchange ten 1000s for one 10,000. 	 Draw representations using place value grid.	 Continue from previous work to carry hundreds as well as tens. Relate to money and measures..
Y5—add numbers with more than 4 digits. Add decimals with 2 decimal places, including money.	As Year 4  Introduce decimal place value counters and model exchange for addition.	As Year 4 	
Y6—add several numbers of increasing complexity, including adding money, measure & decimals with different numbers of decimal points.	As Year 5	As Year 5	 Insert zeros for place holders. 

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Subtraction

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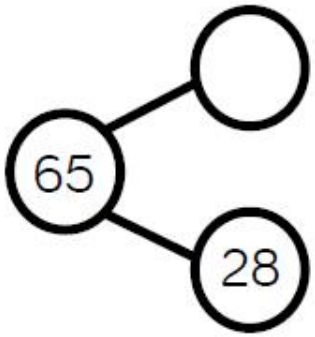
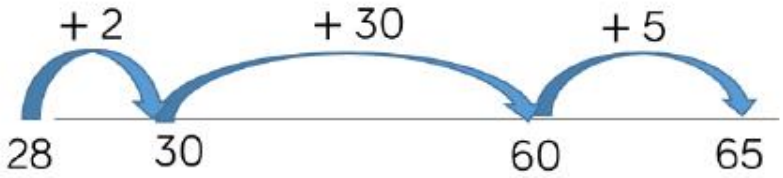
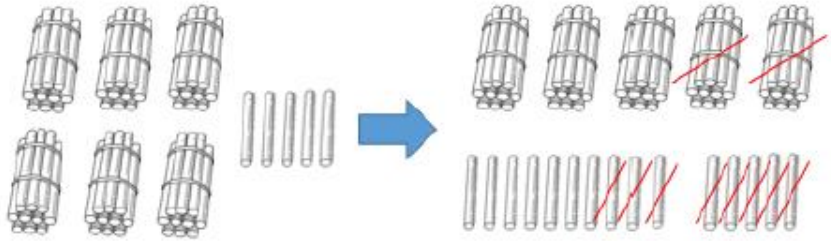
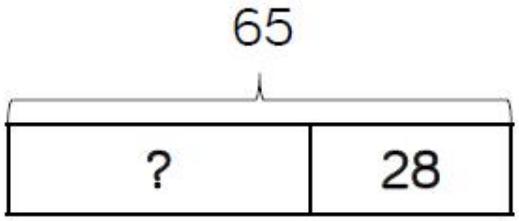
Skill: Subtract 1 and 2-digit numbers to 100	Year: 2/3

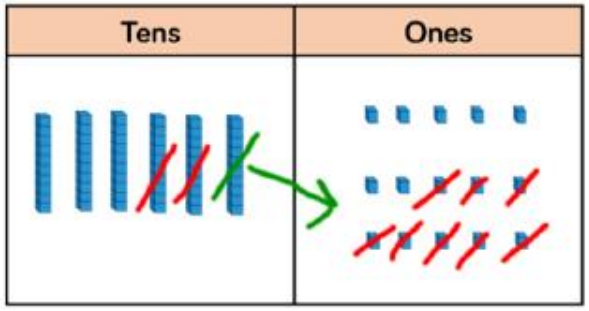




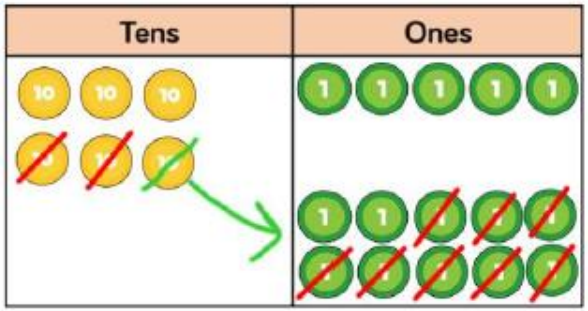




$$65 - 28 = 37$$



$$\begin{array}{r} {}^{5}\cancel{6}{}^{1}5 \\ -\ 28 \\ \hline 37 \end{array}$$



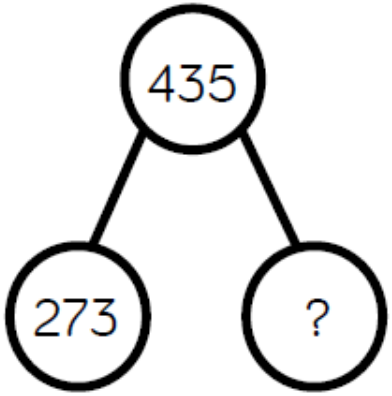
Children can also use a blank number line to count back to find the difference. Encourage them to jump to multiples of 10 to become more efficient.

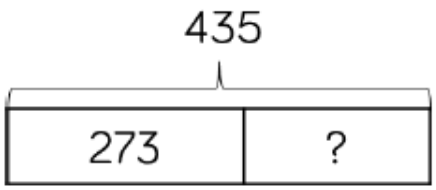
From Year 3, encourage children to use the formal column method when calculating alongside straws, base 10 or place value counters. As numbers become larger, straws become less efficient.

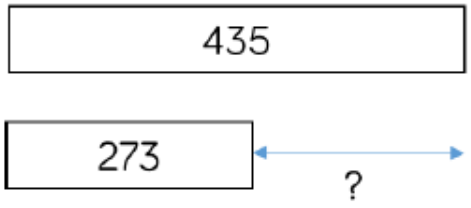
``

Skill: Subtract numbers with up to 3 digits

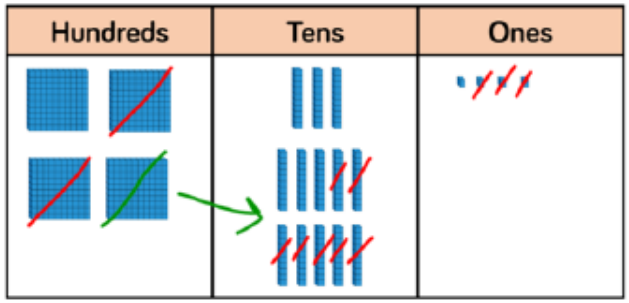
Year: 3



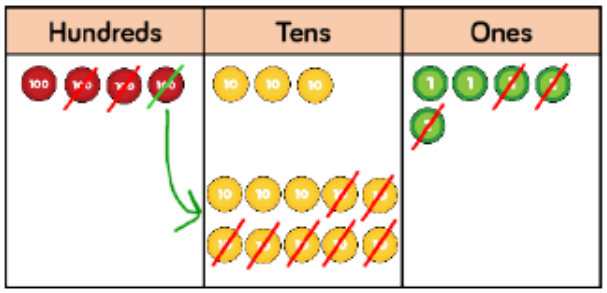




$$435 - 273 = 162$$



$$\begin{array}{r} {}^{3}\not{4}{}^{1}35 \\ -\ 273 \\ \hline 162 \end{array}$$

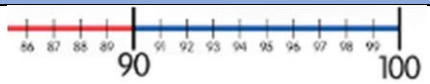
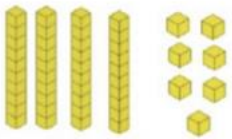
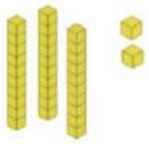
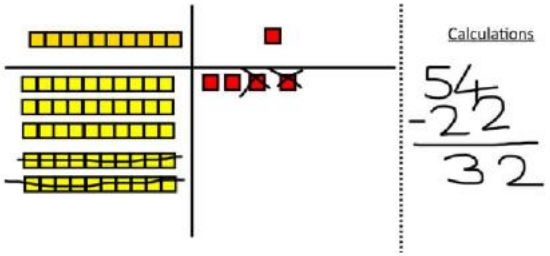
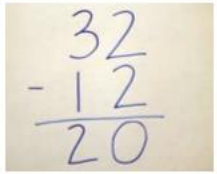


Base 10 and place value counters are the most effective manipulative when subtracting numbers with up to 3 digits.

Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

Plain counters on a place value grid can also be used to support learning.

``

Year 3 - Subtraction			
Objective/ Strategy	Concrete	Pictorial	Abstract
Subtract numbers mentally, including: three-digit number + ones three-digit number + tens three-digit number + hundreds			Expose children to missing number questions and vary the missing part of the calculation. 678=? -1 688 – 10 = ? 678 = ? – 100
Column subtraction without regrouping (friendly numbers)	 Use base 10 or Place Value counters to model  47—32	 Draw representations to support understanding	

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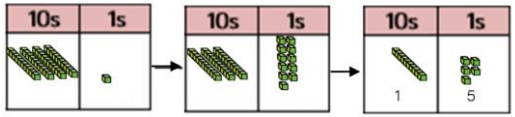
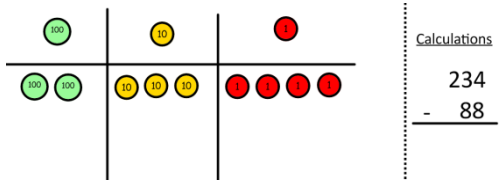
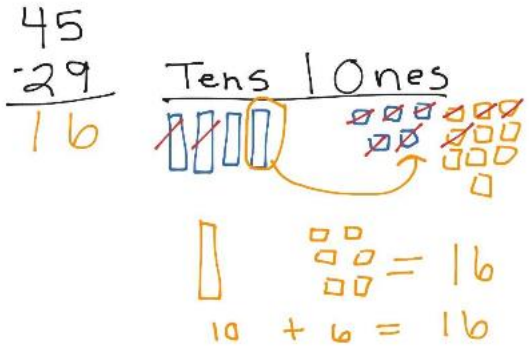
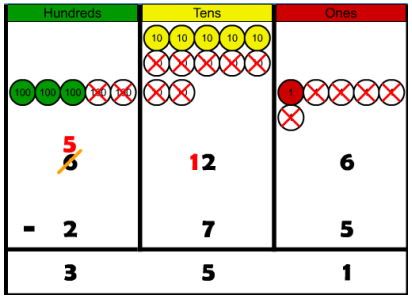
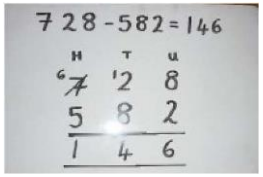
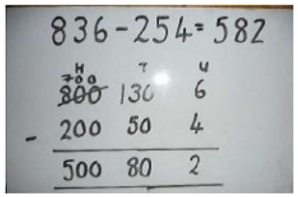
Column subtraction with regrouping	Begin with base 10. Move to pv counters, modelling the exchange of a ten into ten ones. **Column method** using base 10 and having to exchange. 41 – 26  Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges. Make the larger number with the place value counters, removing the counters when subtracting.  Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.	 Children may draw base ten or PV counters and cross off.  Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make When confident, children can find their own way to record the exchange/regrouping.	Children can start their formal written method by partitioning the number into clear place value columns  Moving forward the children use a more compact method. This will lead to an understanding of subtracting any number including decimals. 5 12 1 2 6 3 . 0 - 2 6 . 5 2 3 6 . 5 If needed: Begin by partitioning into pv columns  Then move to formal method.

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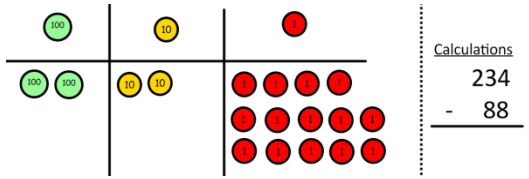
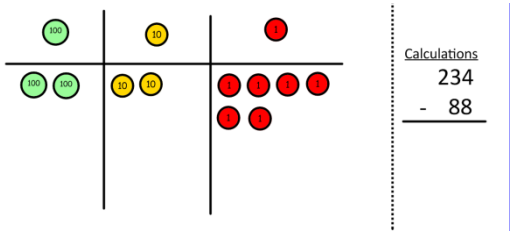
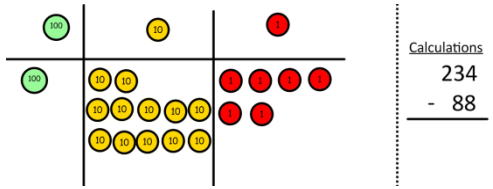


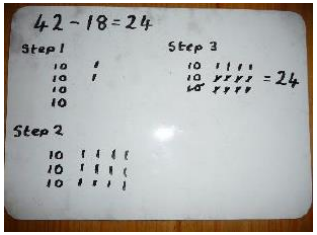
Now I can subtract my ones.



Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.



ow I can take away eight tens and complete my subtraction



Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.

Children could draw plain counters on a PV grid to aid them.

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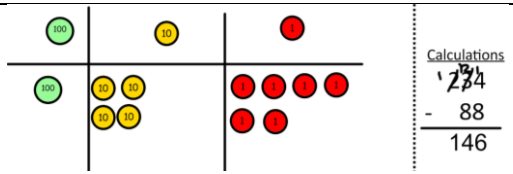
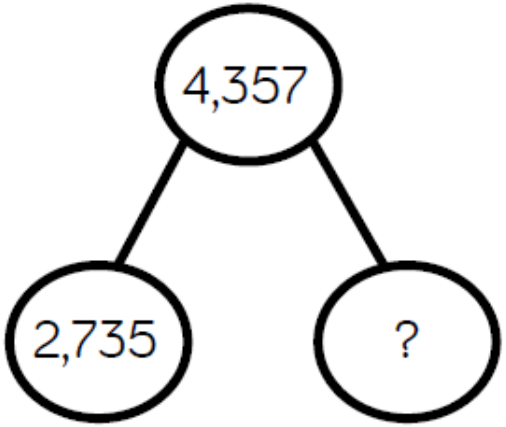
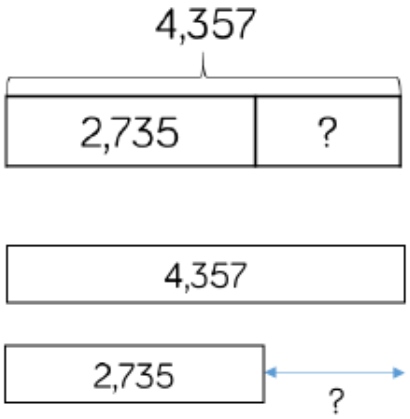
	 Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.		

``

Skill: Subtract numbers with up to 4 digits

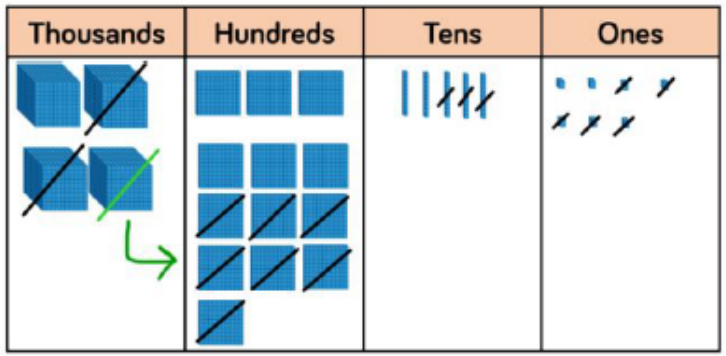
Year: 4

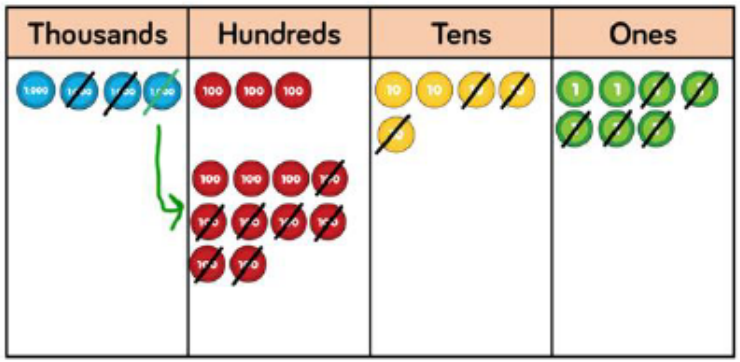




$$\begin{array}{r} {}^{3}\cancel{4}{}^{1}357 \\ -\ 2735 \\ \hline 1622 \end{array}$$

$$4{,}357 - 2{,}735 = 1{,}622$$





Base 10 and place value counters are the most effective manipulatives when subtracting numbers with up to 4 digits.

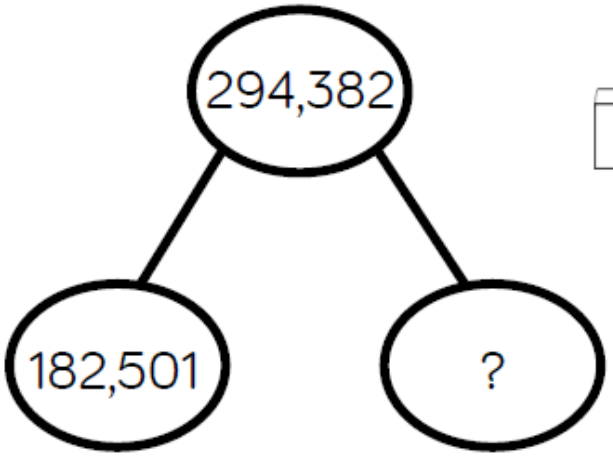
Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

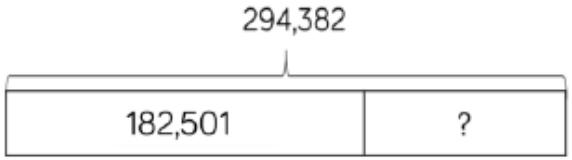
Plain counters on a place value grid can also be used to support learning.

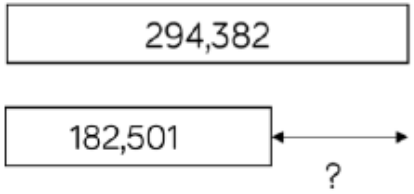
``

Skill: Subtract numbers with more than 4 digits

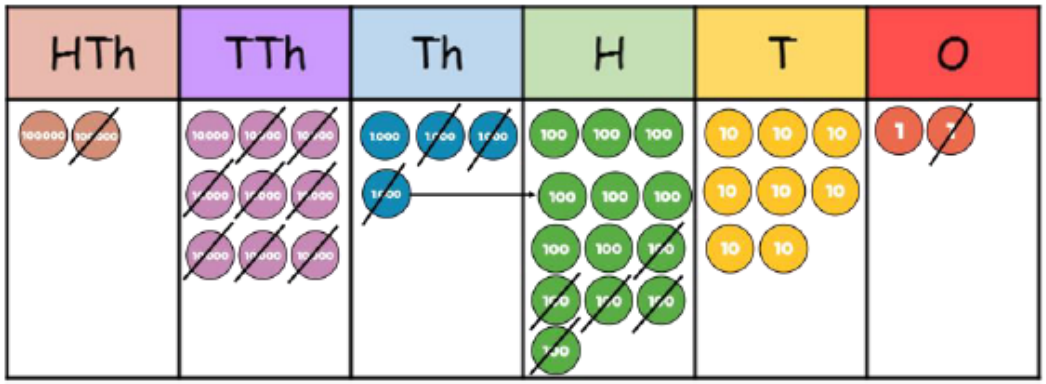
Year: 5/6







294,382 − 182,501 = 111,881



	2	9	$^{3}\not{4}$	$^{1}3$	8	2
−	1	8	2	5	0	1
	1	1	1	8	8	1

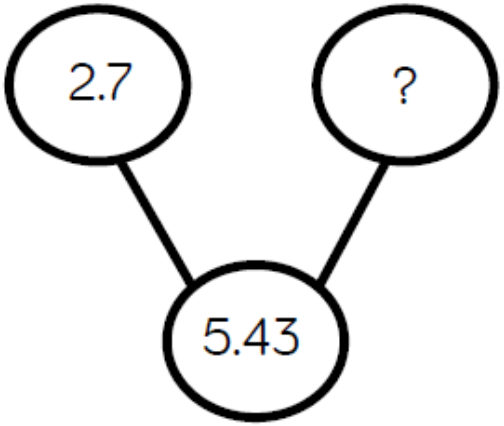
Place value counters or plain counters on a place value grid are the most effective concrete resource when subtracting numbers with more than 4 digits.

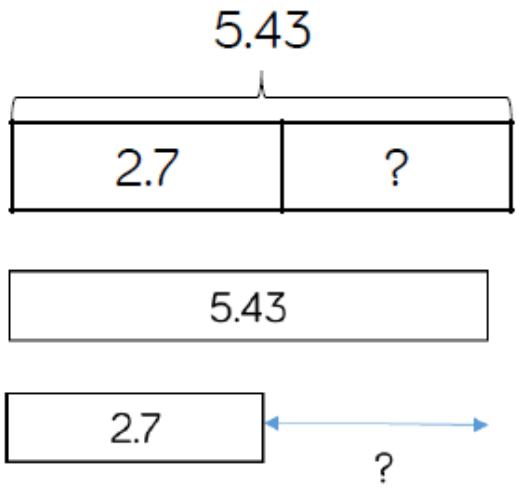
At this stage, children should be encouraged to work in the abstract, using column method to subtract larger numbers efficiently.

``

Skill: Subtract with up to 3 decimal places

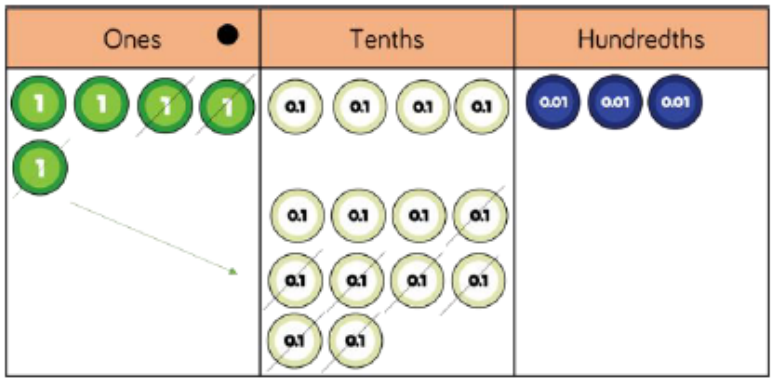
Year: 5/6

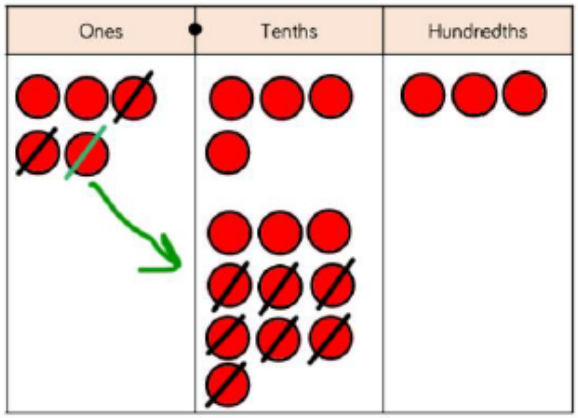




$$\begin{array}{r} {}^{4}\cancel{5}.{}^{1}43 \\ -\ 2.7 \\ \hline 2.73 \end{array}$$

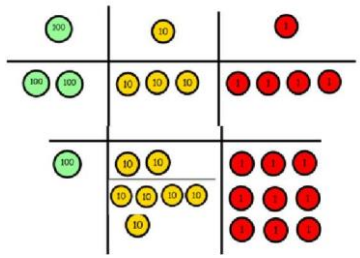
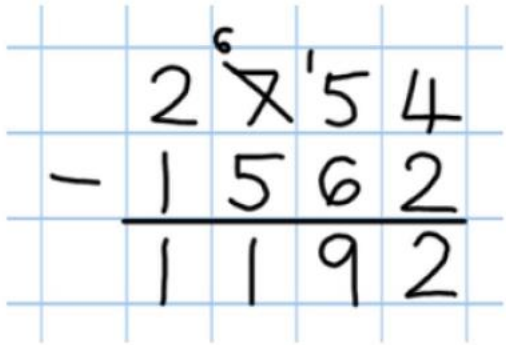
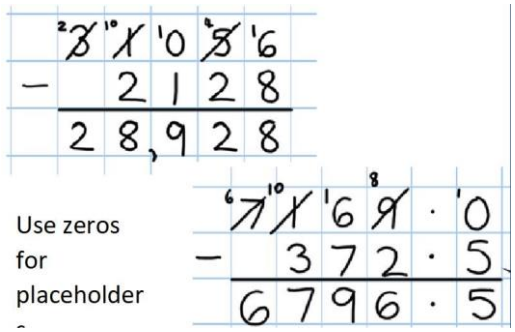
$$5.43 - 2.7 = 2.73$$



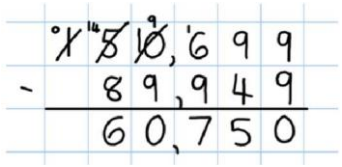
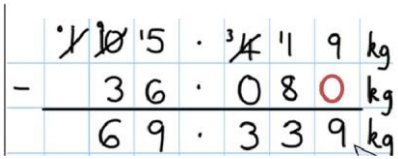


Place value counters and plain counters on a place value grid are the most effective manipulative when subtracting decimals with 1, 2 and then 3 decimal places.

Ensure children have experience of subtracting decimals with a variety of decimal places. This includes putting this into context when subtracting money and other measures.

Year 4 - 6 Subtraction			
Objective/ Strategy	Concrete	Pictorial	Abstract
Subtracting tens and ones Year 4 subtract with up to 4 digits. Introduce decimal subtraction through context of money	 Model process of exchange using, base ten and then move to PV counters.	Children to draw place value counters or Base 10 and show their exchange—see Y3	
Year 5- Subtract with at least 4 digits, including money and measures. Subtract with decimal values, including mixtures of	As Year 4	Children to draw place value counters (or plain on a PV grid) and show their exchange—see Y3	 Use zeros for placeholder

``

integers and decimals and aligning the decimal Up to 3 decimal places			
Year 6— Subtract with increasingly large and more complex numbers and decimal values (up to 3 decimal place).	As Year 4	Children to draw pv counters and show their exchange—see Y3	  Use zero as a place value holder where needed.

``

Glossary

Addend - A number to be added to another.

Aggregation - combining two or more quantities or measures to find a total.

Augmentation - increasing a quantity or measure by another quantity.

Commutative – numbers can be added in any order.

Complement – in addition, a number and its complement make a total e.g. 300 is the complement to 700 to make 1,000

Difference – the numerical difference between two numbers is found by comparing the quantity in each group.

Exchange – Change a number or expression for another of an equal value.

Minuend – A quantity or number from which another is subtracted.

Partitioning – Splitting a number into its component parts.

Reduction – Subtraction as take away.

Subitise – Instantly recognise the number of objects in a small group without needing to count.

Subtrahend - A number to be subtracted from another.

Sum - The result of an addition.

Total – The aggregate or the sum found by addition.

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Multiplication

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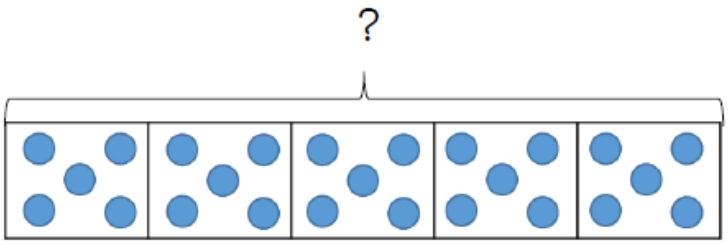
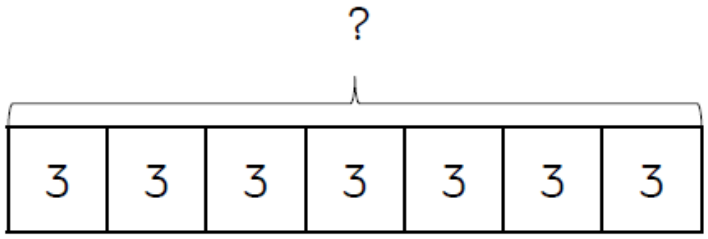
Bar Model

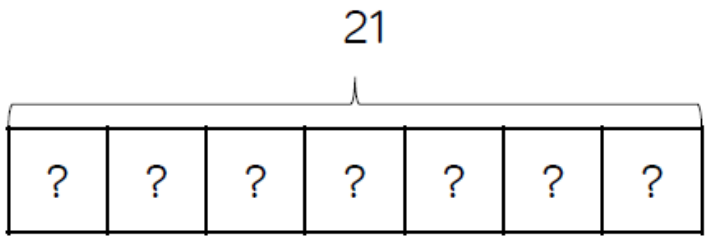


$5 \times 5 = 25$

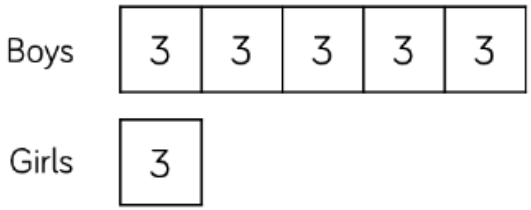


$3 \times 7 = 21$

$7 \times 3 = 21$



$21 \div 7 = 3$



Benefits

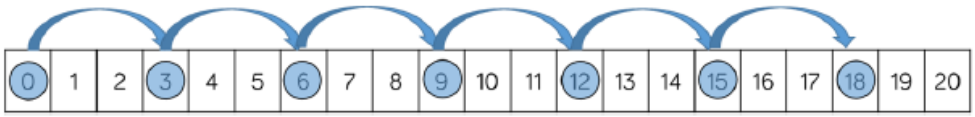
Children can use the single bar model to represent multiplication as repeated addition. They could use counters, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent the multiplication.

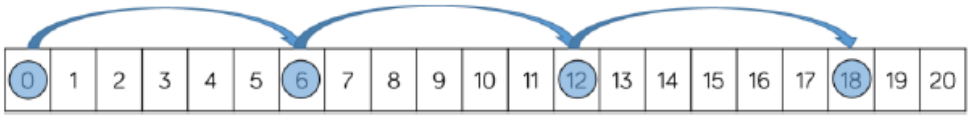
Division can be represented by showing the total of the bar model and then dividing the bar model into equal groups.

It is important when solving word problems that the bar model represents the problem.

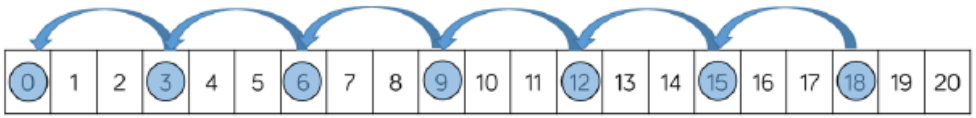
Sometimes, children may look at scaling problems. In this case, more than one bar model is useful to represent this type of problem, e.g. There are 3 girls in a group. There are 5 times more boys than girls. How many boys are there?
The multiple bar model provides an opportunity to compare the groups.

Number Tracks





$6 \times 3 = 18$
$3 \times 6 = 18$



$18 \div 3 = 6$

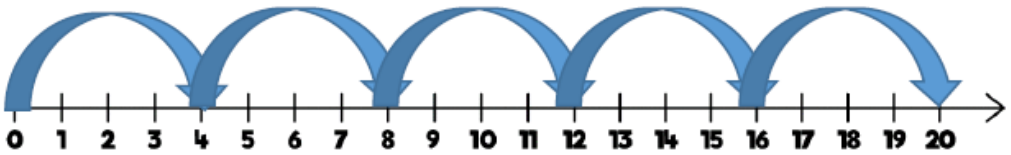
Benefits

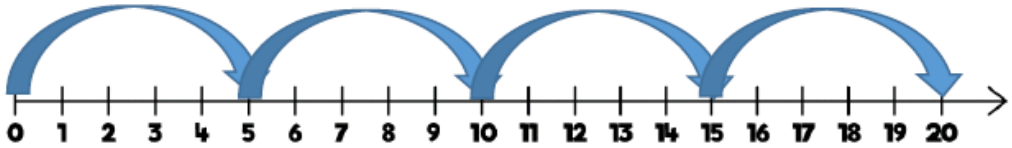
Number tracks are useful to support children to count in multiples, forwards and backwards. Moving counters or cubes along the number track can support children to keep track of their counting. Translucent counters help children to see the number they have landed on whilst counting.

When multiplying, children place their counter on 0 to start and then count on to find the product of the numbers.
When dividing, children place their counter on the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0. Children record how many jumps they have made to find the answer to the division.

Number tracks can be useful with smaller multiples but when reaching larger numbers they can become less efficient.

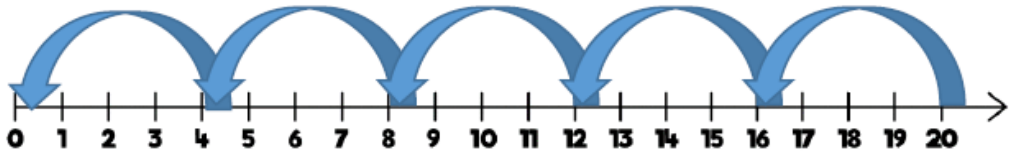
Number Lines (labelled)





$4 \times 5 = 20$

$5 \times 4 = 20$



$20 \div 4 = 5$

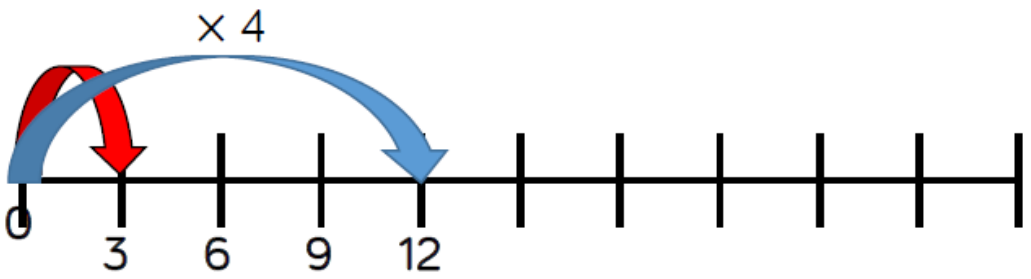
Benefits

Labelled number lines are useful to support children to count in multiples, forwards and backwards as well as calculating single-digit multiplications.

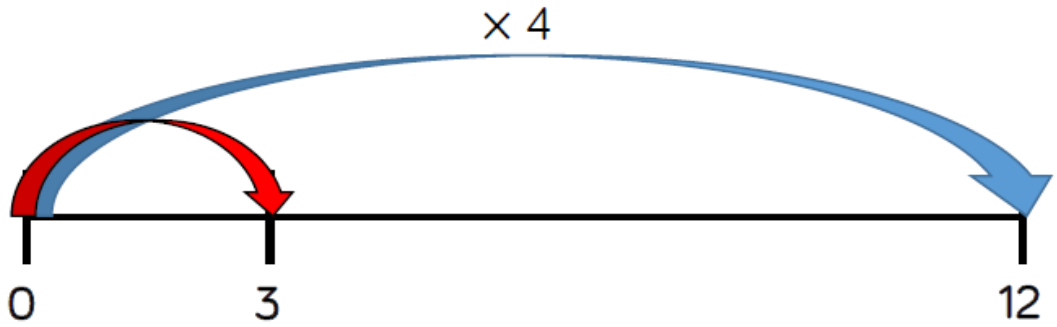
When multiplying, children start at 0 and then count on to find the product of the numbers.
When dividing, start at the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0.
Children record how many jumps they have made to find the answer to the division.

Labelled number lines can be useful with smaller multiples, however they become inefficient as numbers become larger due to the required size of the number line.

Number Lines (blank)



A red car travels 3 miles.
A blue car 4 times further.
How far does the blue car travel?



A blue car travels 12 miles.
A red car 4 times less.
How far does the red car travel?

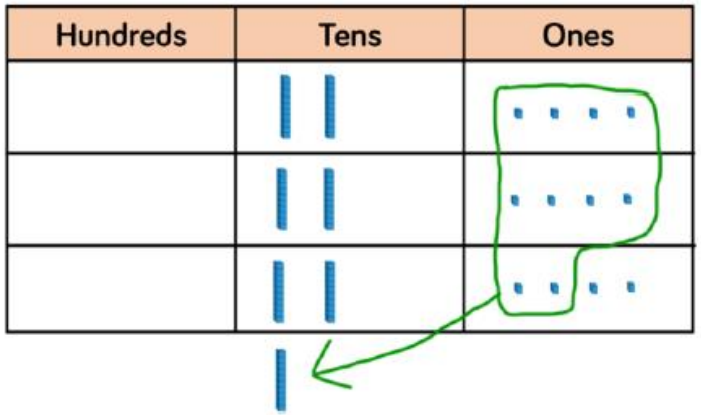
Benefits

Children can use blank number lines to represent scaling as multiplication or division.

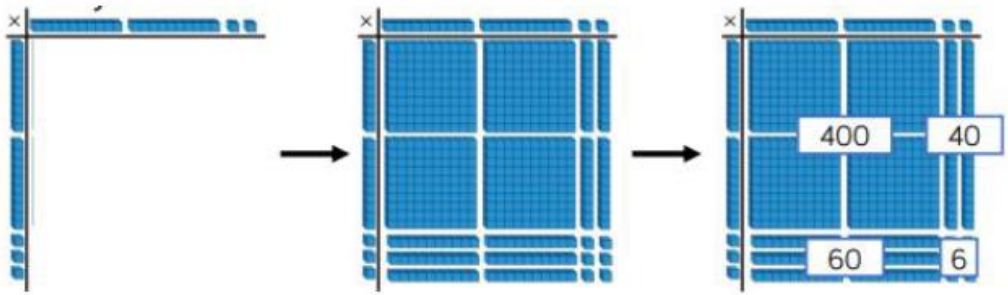
Blank number lines with intervals can support children to represent scaling accurately. Children can label intervals with multiples to calculate scaling problems.

Blank number lines without intervals can also be used for children to represent scaling.

Base 10/Dienes (multiplication)



$$\begin{array}{r} 24 \\ \times\ 3 \\ \hline 72 \\ \hline 1 \end{array}$$



Benefits

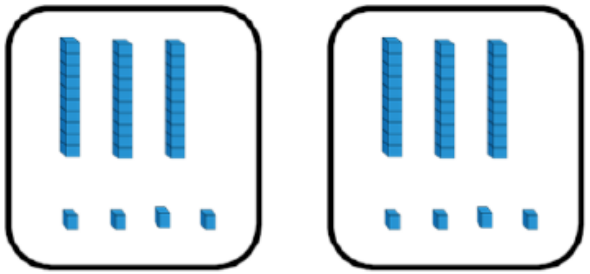
Using Base 10 or Dienes is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written representations match.

As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed.

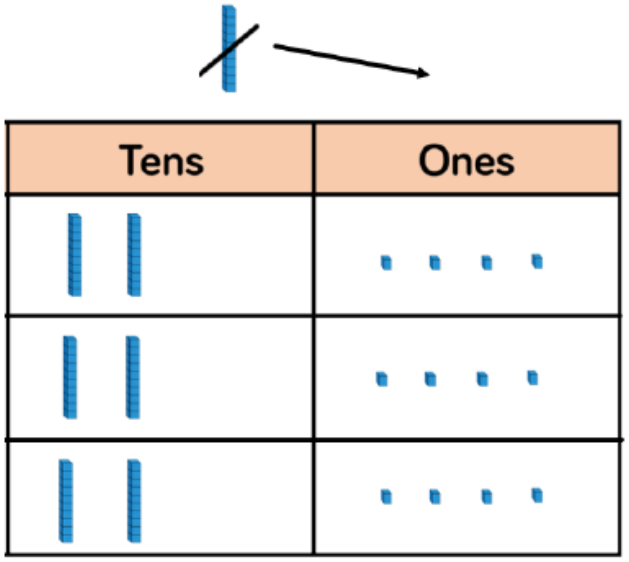
Base 10 also supports the area model of multiplication well. Children use the equipment to build the number in a rectangular shape which they then find the area of by calculating the total value of the pieces This area model can be linked to the grid method or the formal column method of multiplying 2-digits by 2-digits.

``

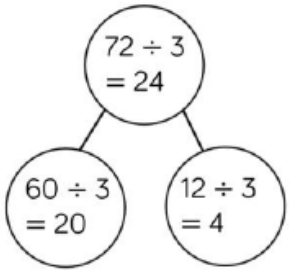
Base 10/Dienes (division)



$68 \div 2 = 34$



$72 \div 3 = 24$



Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of division.

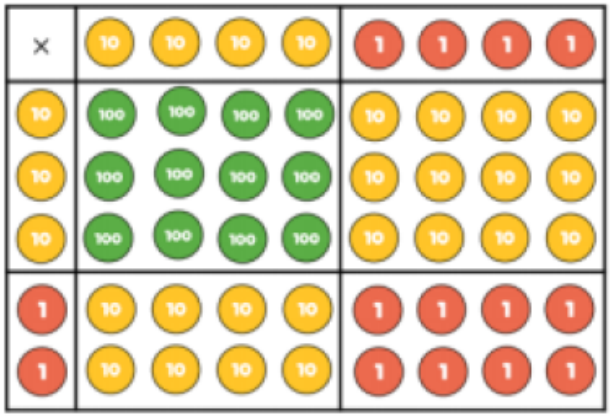
When numbers become larger, it can be an effective way to move children from representing numbers as ones towards representing them as tens and ones in order to divide. Children can then share the Base 10/ Dienes between different groups e.g. by drawing circles or by rows on a place value grid.

When they are sharing, children start with the larger place value and work from left to right. If there are any left in a column, they exchange e.g. one ten for ten ones. When recording, encourage children to use the part-whole model so they can consider how the number has been partitioned in order to divide. This will support them with mental methods.

``

Place Value Counters (multiplication)

$$\begin{array}{r} 34 \\ \times\ 5 \\ \hline 170 \\ \hline 1\ 2 \end{array}$$



$$\begin{array}{r} 44 \\ \times\ 32 \\ \hline 8 \\ 80 \\ 120 \\ +\ 1200 \\ \hline 1408 \\ \hline 1 \end{array}$$

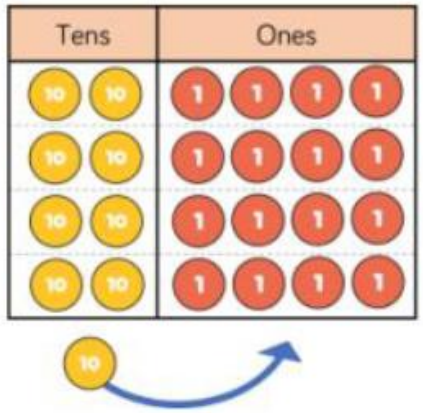
Benefits

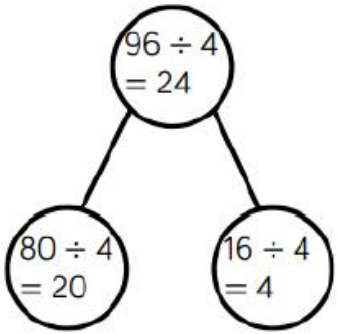
Using place value counters is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written match.

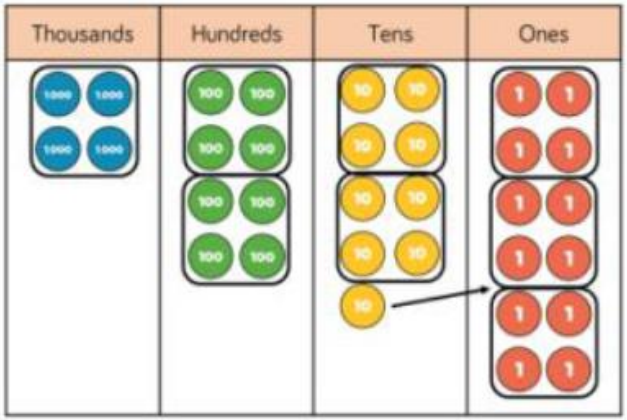
As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed The counters should be used to support the understanding of the written method rather than support the arithmetic.

Place value counters also support the area model of multiplication well. Children can see how to multiply 2-digit numbers by 2-digit numbers.

Place Value Counters (division)







$$\begin{array}{r} 1223 \\ 4\overline{)\,4892} \end{array}$$

Benefits

Using place value counters is an effective way to support children's understanding of division.

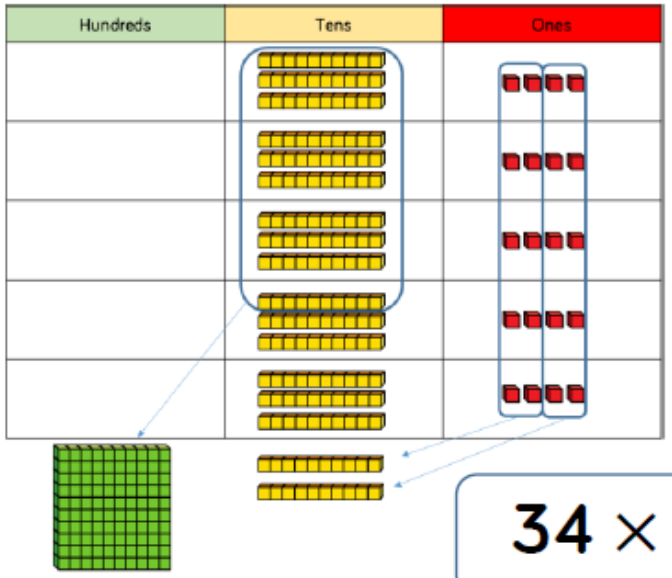
When working with smaller numbers, children can use place value counters to share between groups. They start by sharing the larger place value column and work from left to right. If there are any counters left over once they have been shared, they exchange the counter e.g. exchange one ten for ten ones. This method can be linked to the part-whole model to support children to show their thinking.

Place value counters also support children's understanding of short division by grouping the counters rather than sharing them. Children work from left to right through the place value columns and group the counters in the number they are dividing by. If there are any counters left over after they have been grouped, they exchange the counter e.g. exchange one hundred for ten tens.

``

Skill: Multiply 2-digit numbers by 1-digit numbers

Year: 3/4

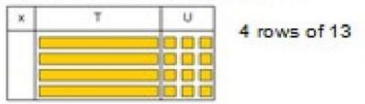
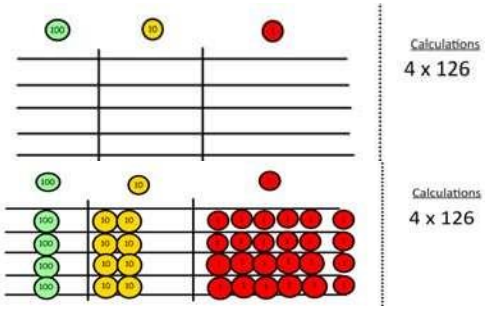
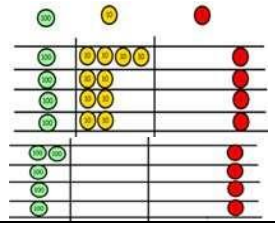
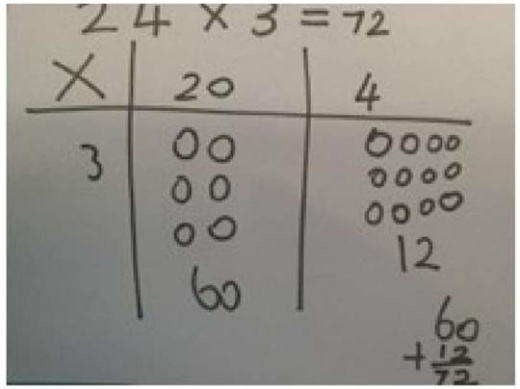
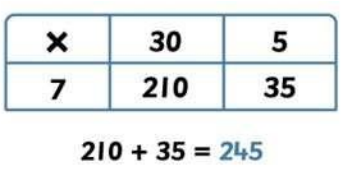


	H	T	O	
		3	4	
×			5	
		2	0	(5 × 4)
+	1	5	0	(5 × 30)
	1	7	0	

$$34 \times 5 = 170$$

	H	T	O	
		3	4	
×			5	
	1	7	0	
	1	2		

Informal methods and the expanded method are used in Year 3 before moving on to the short multiplication method in Year 4.
Place value counters should be used to support the understanding of the method rather than supporting the multiplication, as children should use times table knowledge.

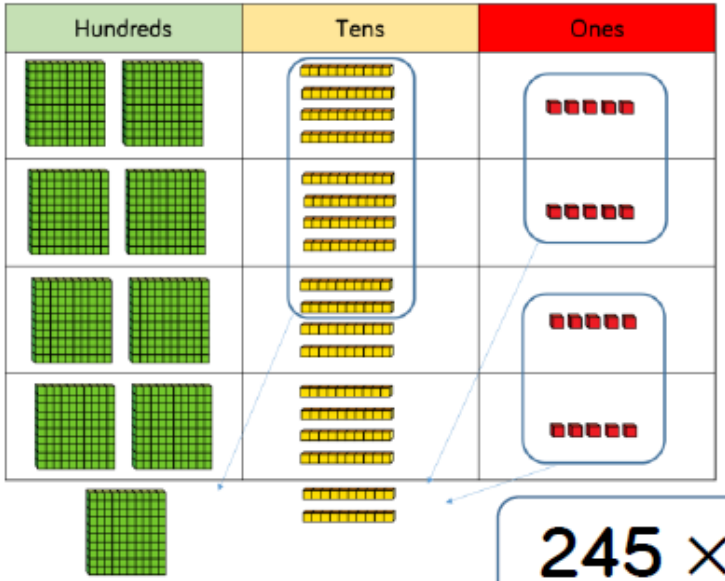
Year 3- Multiplication			
Objective/ Strategy	Concrete	Pictorial	Abstract
Grid method, progressing to the formal method Multiply 2-digit numbers by 1-digit numbers	Grid method - show base ten to move towards a more compact method. (This is taught at secondary so we do need to expose the children to this)  Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows  Fill each row with 126. Add up each column, starting with the ones making any exchanges needed Then you have your answer. 	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.  	Start with multiplying by one digit numbers and showing the clear addition alongside the grid.  Move forward to the formal written method: 35 X 7 245 3 6 x 23 = 23 × 6 138 1 1 **Carry both at the top and bottom.**

``

``

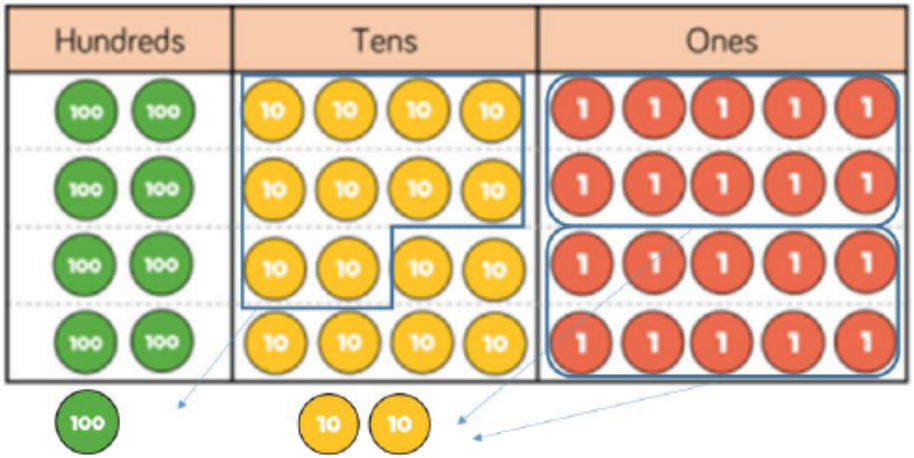
Skill: Multiply 3-digit numbers by 1-digit numbers

Year: 4



	H	T	O
	2	4	5
×			4
	9	8	0
	1	2	

$$245 \times 4 = 980$$



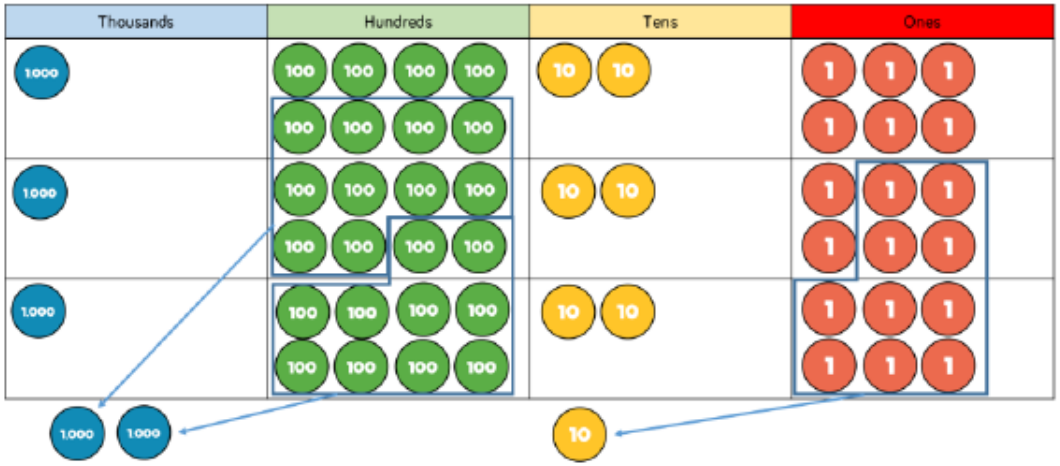
When moving to 3-digit by 1-digit multiplication, encourage children to move towards the short, formal written method.

Base 10 and place value counters continue to support the understanding of the written method.

Limit the number of exchanges needed in the questions and move children away from resources when multiplying larger numbers.

Skill: Multiply 4-digit numbers by 1-digit numbers

Year: 5



$$1{,}826 \times 3 = 5{,}478$$

	Th	H	T	O
	1	8	2	6
×				3
	5	4	7	8
	2		1	

When multiplying 4-digit numbers, place value counters are the best manipulative to use to support children in their understanding of the formal written method.
If children are multiplying larger numbers and struggling with their times tables, encourage the use of multiplication grids so children can focus on the use of the written method.

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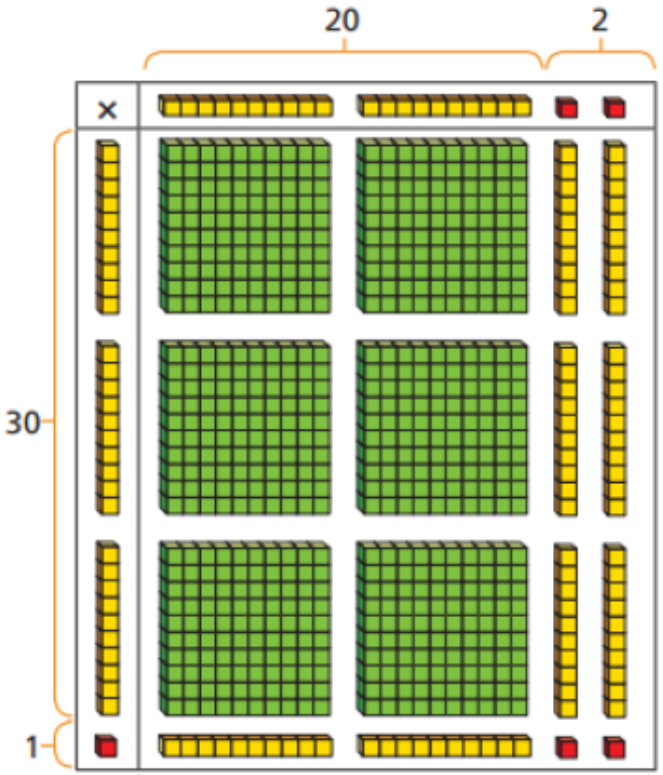
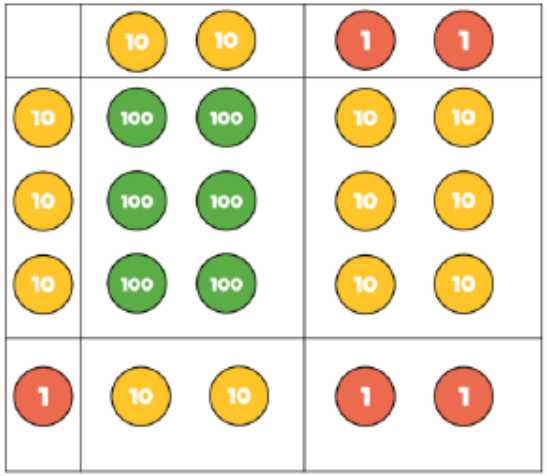
Skill: Multiply 2-digit numbers by 2-digit numbers

Year: 5





×	20	2
30	600	60
1	20	2

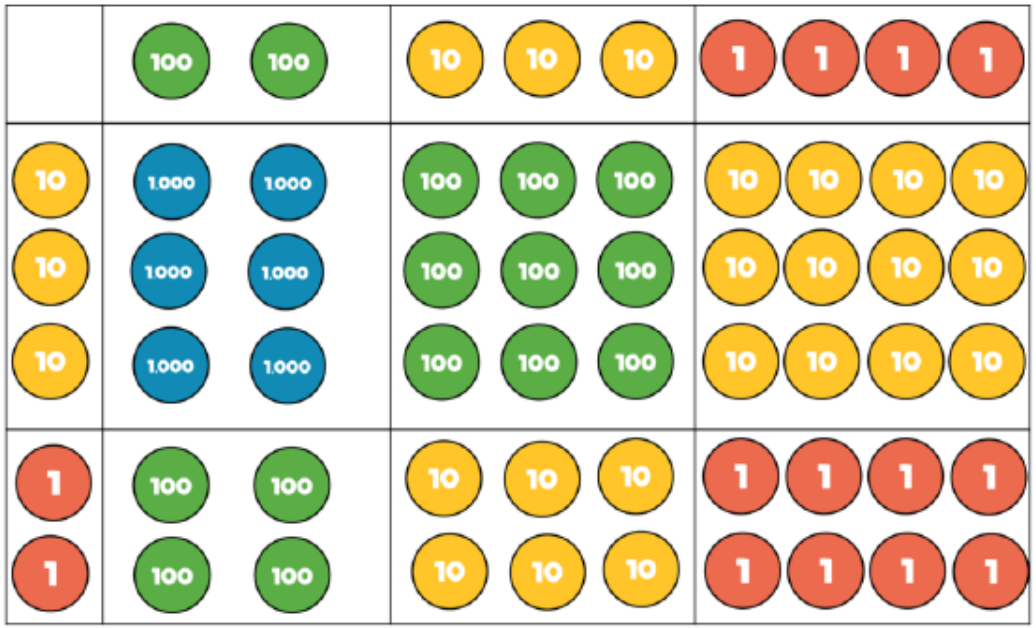
	H	T	O
		2	2
×		3	1
		2	2
	6	6	0
	6	8	2

$$22 \times 31 = 682$$

When multiplying a multi-digit number by 2-digits, use the area model to help children understand the size of the numbers they are using. This links to finding the area of a rectangle by finding the space covered by the Base 10.
The grid method matches the area model as an initial written method before moving on to the formal written multiplication method.

Skill: Multiply 3-digit numbers by 2-digit numbers

Year: 5



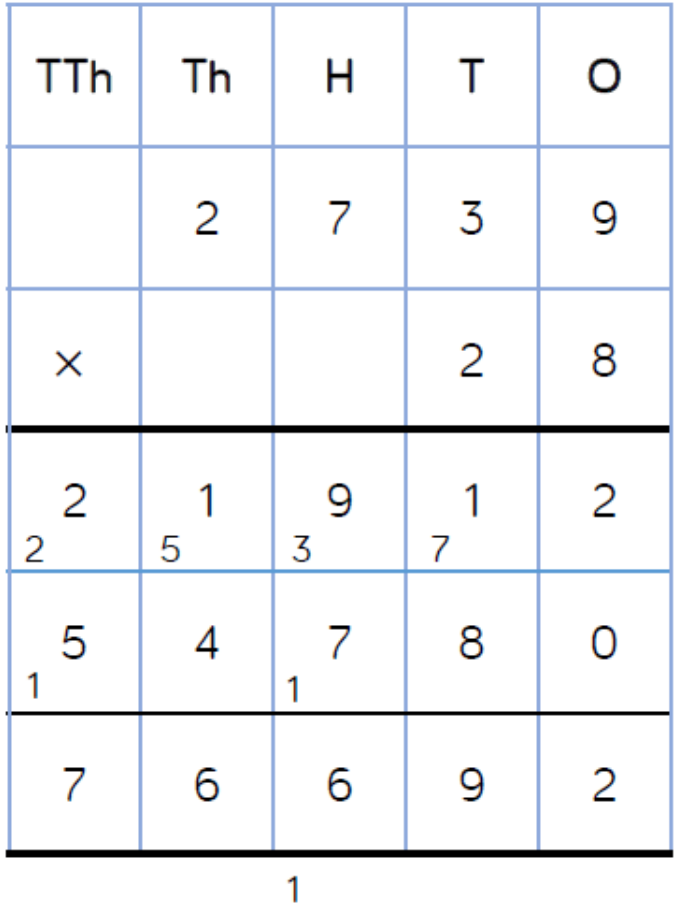
Th	H	T	O
	2	3	4
×		3	2
	4	6	8
$^{1}7$	$^{1}0$	2	0
7	4	8	8

×	200	30	4
30	6,000	900	120
2	400	60	8

$$234 \times 32 = 7{,}488$$

Children can continue to use the area model when multiplying 3-digits by 2-digits. Place value counters become more efficient to use but Base 10 can be used to highlight the size of numbers.

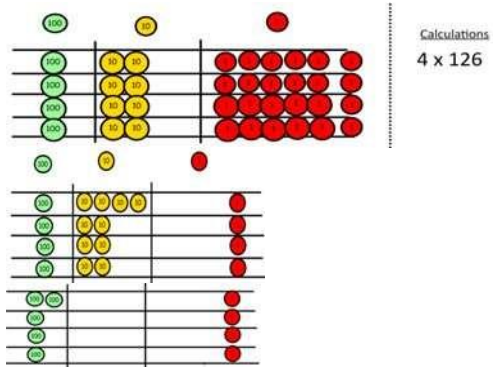
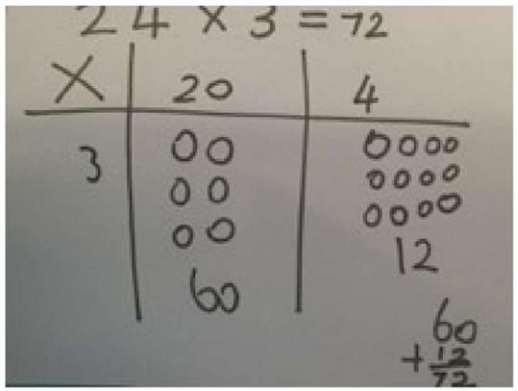
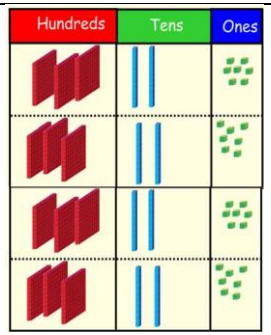
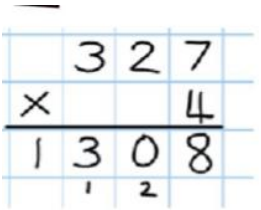
Children should now move towards the formal written method, seeing the links with the grid method.

Skill: Multiply 4-digit numbers by 2-digit numbers	Year: 5/6
	When multiplying 4-digits by 2-digits, children should be confident in using the formal written method. If they are still struggling with times tables, provide multiplication grids to support when they are focusing on the use of the method. Consider where exchanged digits are placed and make sure this is consistent.

TTh	Th	H	T	O
	2	7	3	9
×			2	8
2 (2)	1 (5)	9 (3)	1 (7)	2
5 (1)	4	7 (1)	8	0
7	6	6	9	2
		1		

$$2{,}739 \times 28 = 76{,}692$$

``

Year 4-6 Multiplication			
Objective/ Strategy	Concrete	Pictorial	Abstract
Grid method recap if necessary	Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.  Add up each column, starting with the ones making any exchanges needed	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.  	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. | × | 30 | 5 | | 7 | 210 | 35 | 210 + 35 = 245
Column Multiplication	 Children can continue to be	| x | 300 | 20 | 7 | | 4 | 1200 | 80 | 28 | The grid method may be used to show how this relates to a formal written method. This is taught at secondary so they do need to be exposed to this method.	 This may lead to a compact method.

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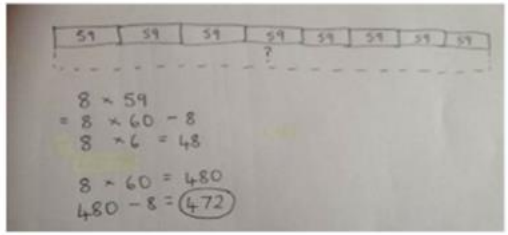
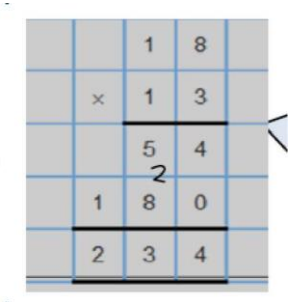
	supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 327 x 4 = 1308 It is important at this stage that they always Multiply the ones first. The corresponding long multiplication is modelled alongside.	 Bar modelling can support learners when solving problems with multiplication alongside the formal written methods.	
Column Multiplication for 3- and 4-digits x 1 digit.	As above	As above. Times tables grids to support with the method and to take focus off times tables recall.	As above
Column multiplication	Manipulatives may still be used with the corresponding long multiplication modelled alongside.	Drawing of manipulatives may still be used with the corresponding long multiplication modelled alongside.	 18 x 3 on the first row (8 x 3 =24, carrying the 2 for 20, then 1 x 3) 18 x 10 on the 2nd row. Show multiplying by 10 by putting zero in ones first. **Carry at the top or bottom (BOTH)**

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<table>
<tr>
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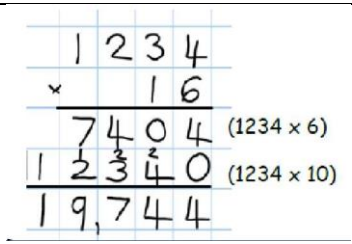
<td></td>
<td> 
</td>
</tr>
<tr>
<td>Multiplying decimals up to 2 decimal places by a single digit</td>
<td>Use previous methods taught.</td>
<td>Use previous methods taught.</td>
<td>Emphasise that the 8 does not represent 0.08 – it is 8 lots of 3.19 (avoids misconceptions regarding place value)

Could use grid method here to prepare children for secondary school.

<table>
<tr><td></td><td></td><td>3</td><td>.</td><td>1</td><td>9</td></tr>
<tr><td>x</td><td></td><td></td><td></td><td></td><td>8</td></tr>
<tr><td></td><td>2</td><td>5</td><td>.</td><td>5</td><td>2</td></tr>
<tr><td></td><td></td><td>1</td><td></td><td>7</td><td></td></tr>
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Division

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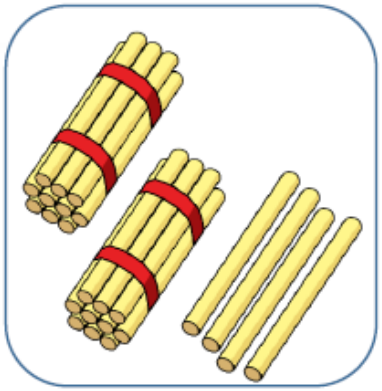
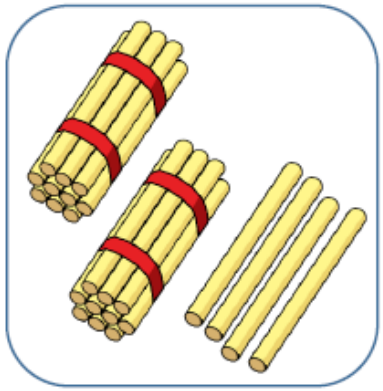
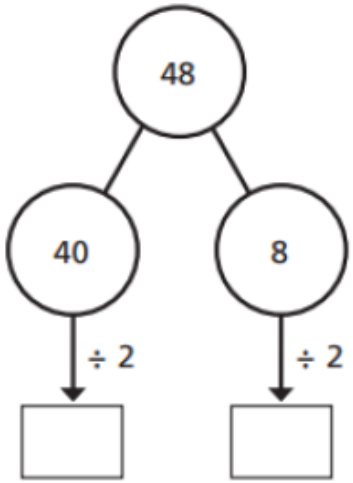
Skill: Divide 2-digits by 1-digit (sharing with no exchange)

Year: 3

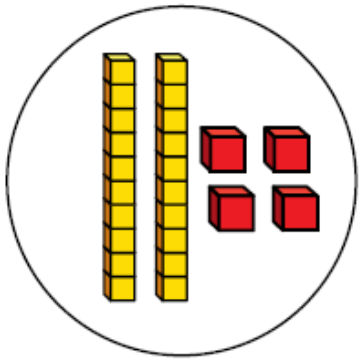
Tens	Ones
10 10	1 1 1 1
10 10	1 1 1 1

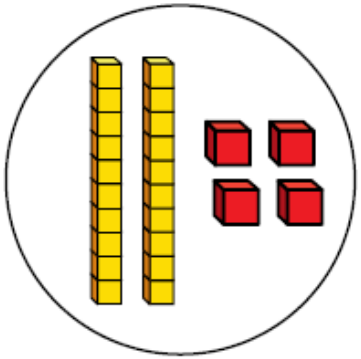






$$48 \div 2 = 24$$





When dividing larger numbers, children can use manipulatives that allow them to partition into tens and ones.

Straws, Base 10 and place value counters can all be used to share numbers into equal groups.

Part-whole models can provide children with a clear written method that matches the concrete representation.

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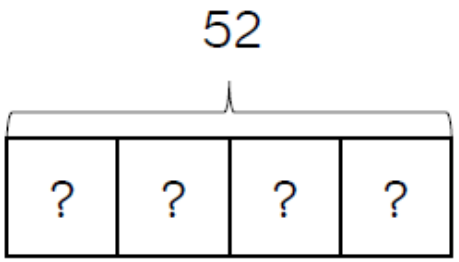
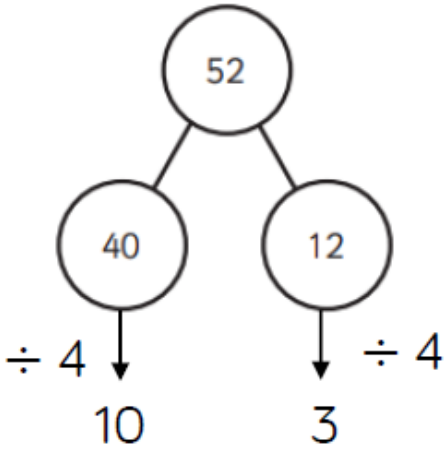
Skill: Divide 2-digits by 1-digit (sharing with exchange)

Year: 3/4

Tens	Ones



$$52 \div 4 = 13$$



$10 + 3 = 13$

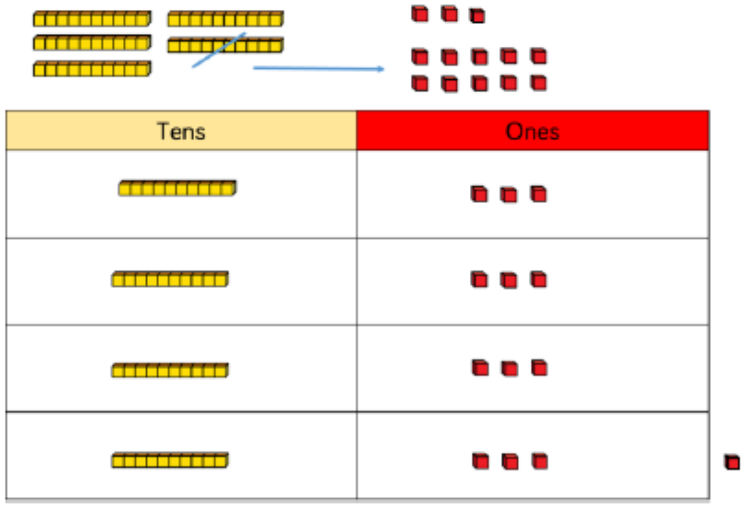
Tens	Ones
10	1 1 1
10	1 1 1
10	1 1 1
10	1 1 1

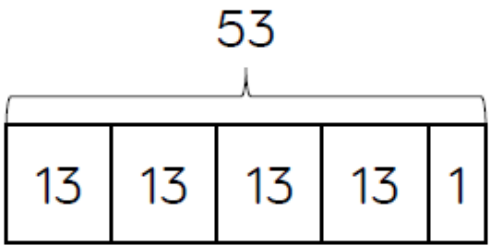
When dividing numbers involving an exchange, children can use Base 10 and place value counters to exchange one ten for ten ones.
Children should start with the equipment outside the place value grid before sharing the tens and ones equally between the rows.

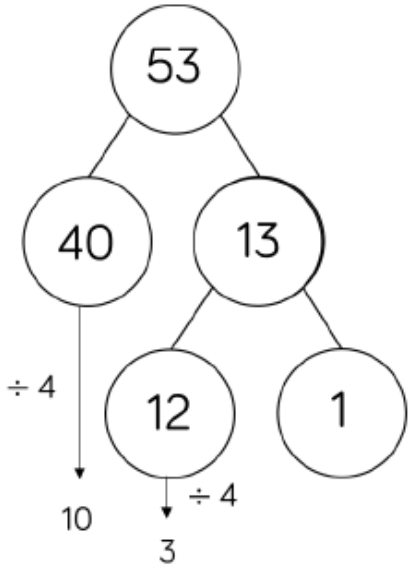
Flexible partitioning in a part-whole model supports this method.

Skill: Divide 2-digits by 1-digit (sharing with remainders)

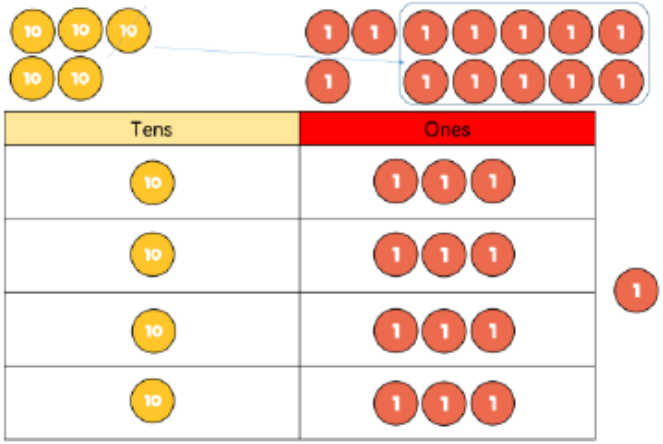
Year: 3/4



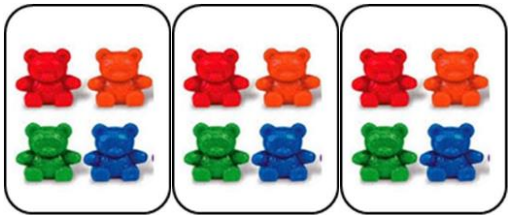
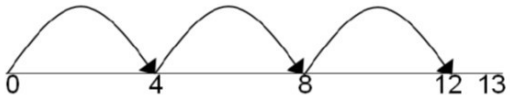
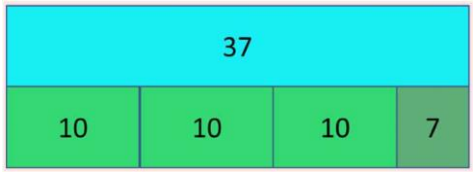




53 ÷ 4 = 13 r1



When dividing numbers with remainders, children can use Base 10 and place value counters to exchange one ten for ten ones.
Starting with the equipment outside the place value grid will highlight remainders, as they will be left outside the grid once the equal groups have been made.
Flexible partitioning in a part-whole model supports this method.

Year 3 - Division			
Objective/ Strategy	Concrete	Pictorial	Abstract
Division with remainders.	14 ÷ 3 = Divide objects between groups and see how much is left over  	then Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  Use bar models to show division with remainders.  	Complete written divisions and show the remainder using r.  Short (bus stop) division for 2 digit numbers: - No exchange (friendly) - Exchanging - Remainders -

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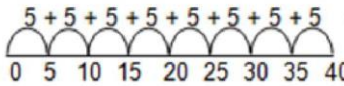
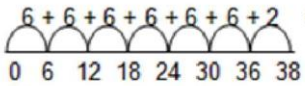
		remainder: 5s in 40?"  mainder:  rs, when it becomes inefficient to count in single mu orded using known facts.	

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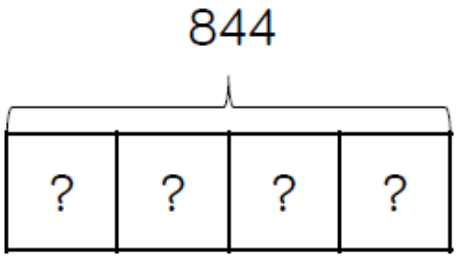
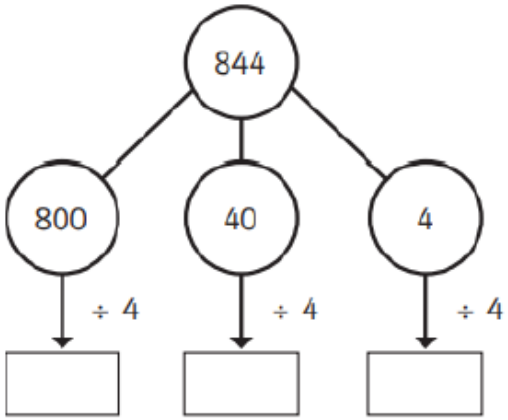
Skill: Divide 3-digits by 1-digit (sharing)

Year: 4

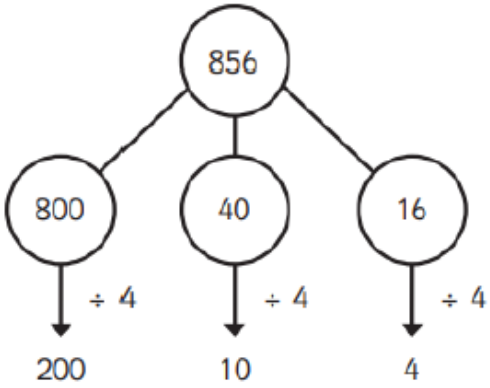
844 ÷ 4 = 211

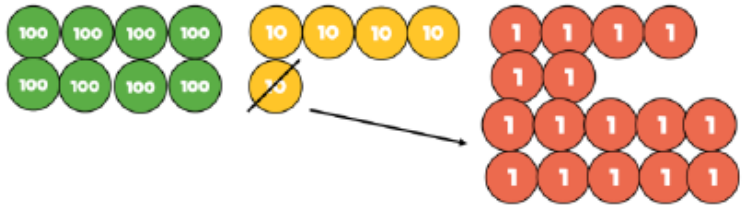


H	T	O
100 100	10	1
100 100	10	1
100 100	10	1
100 100	10	1



856 ÷ 4 = 214





Hundreds	Tens	Ones
100 100	10	1 1 1 1
100 100	10	1 1 1 1
100 100	10	1 1 1 1
100 100	10	1 1 1 1

Children can continue to use place value counters to share 3-digit numbers into equal groups.
Children should start with the equipment outside the place value grid before sharing the hundreds, tens and ones equally between the rows.
This method can also help to highlight remainders.
Flexible partitioning in a part-whole model supports this method.

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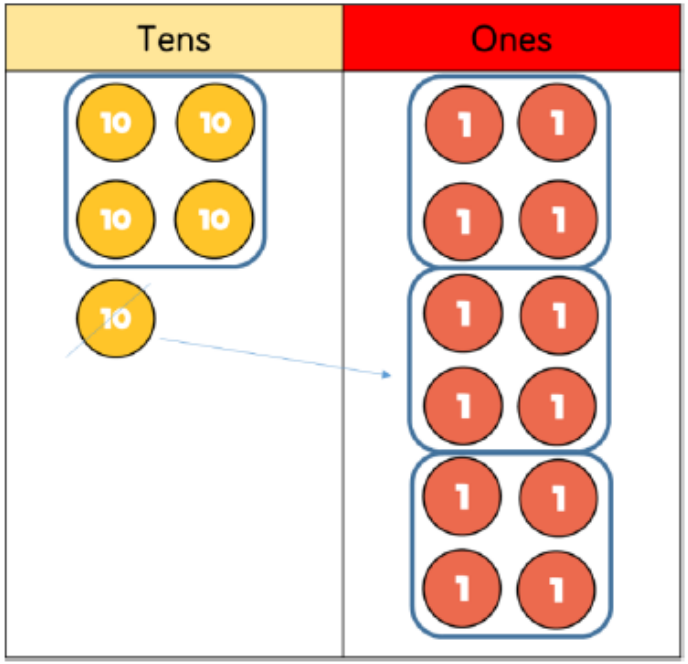
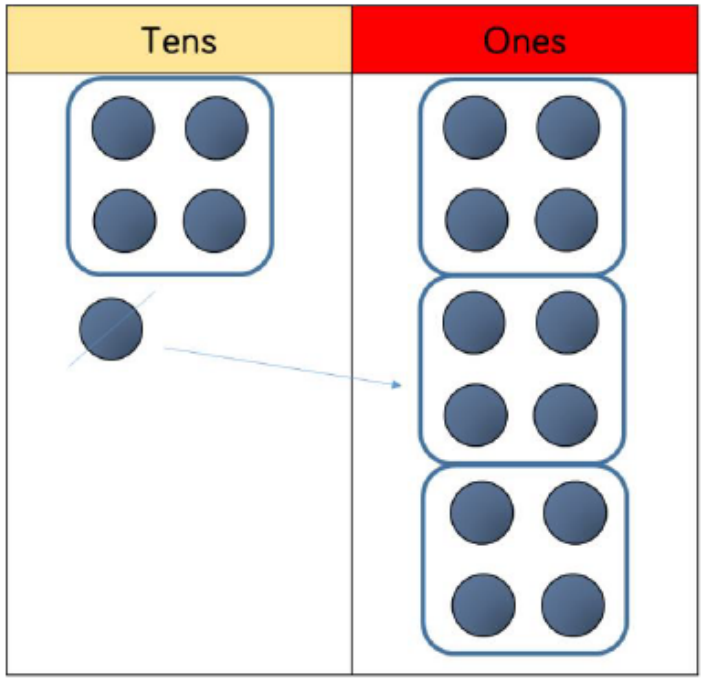
Skill: Divide 2-digits by 1-digit (grouping)

Year: 5



		1	3	
	4	5	1 2	

$$52 \div 4 = 13$$

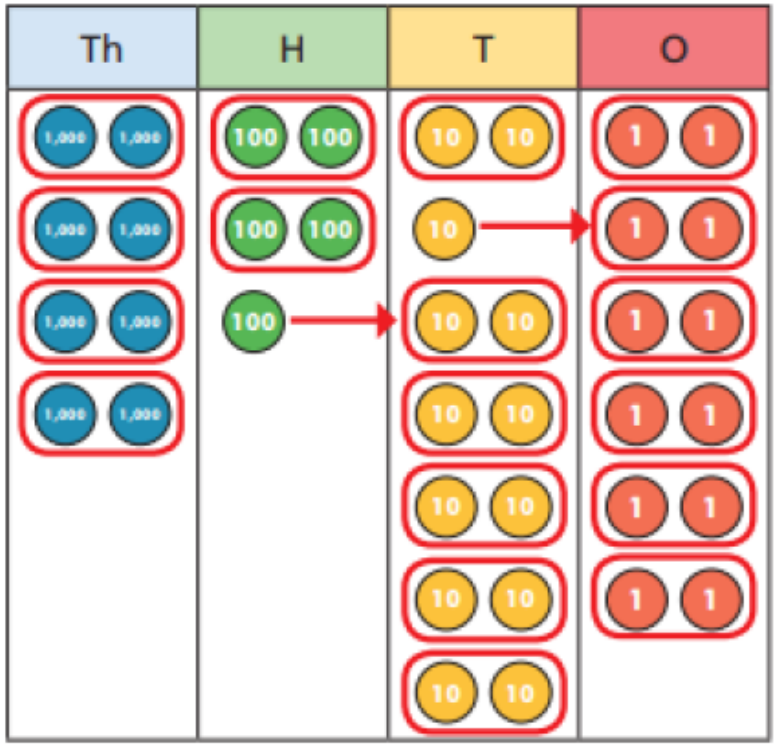


When using the short division method, children use grouping. Starting with the largest place value, they group by the divisor.

Language is important here. Children should consider 'How many groups of 4 tens can we make?' and 'How many groups of 4 ones can we make?'

Remainders can also be seen as they are left ungrouped.

Skill: Divide 4-digits by 1-digit (grouping)	Year: 5



	4	2	6	6
2	8	5	$^{1}3$	$^{1}2$

$$8{,}532 \div 2 = 4{,}266$$

Place value counters or plain counters can be used on a place value grid to support children to divide 4-digits by 1-digit. Children can also draw their own counters and group them through a more pictorial method.

Children should be encouraged to move away from the concrete and pictorial when dividing numbers with multiple exchanges.

``

Skill: Divide multi digits by 2-digits (short division)	Year: 6
	When children begin to divide up to 4-digits by 2-digits, written methods become the most accurate as concrete and pictorial representations become less effective. Children can write out multiples to support their calculations with larger remainders. Children will also solve problems with remainders where the quotient can be rounded as appropriate.

		0	3	6
	12	4	$^{4}3$	$^{7}2$

$$432 \div 12 = 36$$

$$7{,}335 \div 15 = 489$$

	0	4	8	9
15	7	$^{7}3$	$^{13}3$	$^{13}5$

15	30	45	60	75	90	105	120	135	150

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Glossary

Array – An ordered collection of counters, cubes or other item in rows and columns.

Commutative – Numbers can be multiplied in any order.

Dividend – In division, the number that is divided.

Divisor – In division, the number by which another is divided.

Exchange – Change a number or expression for another of an equal value.

Factor – A number that multiplies with another to make a product.

Multiplicand – In multiplication, a number to be multiplied by another.

Partitioning – Splitting a number into its component parts.

Product – The result of multiplying one number by another.

Quotient – The result of a division

Remainder – The amount left over after a division when the divisor is not a factor of the dividend.

Scaling – Enlarging or reducing a number by a given amount, called the scale factor

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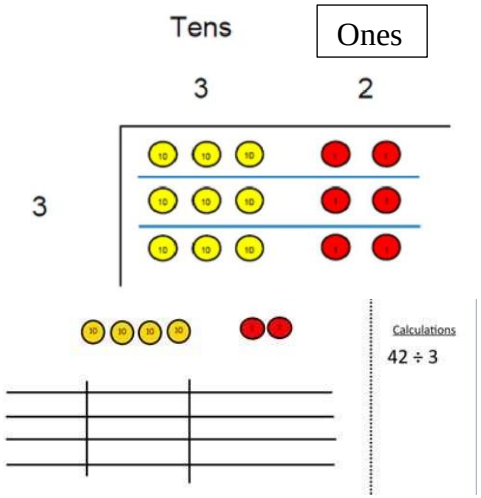
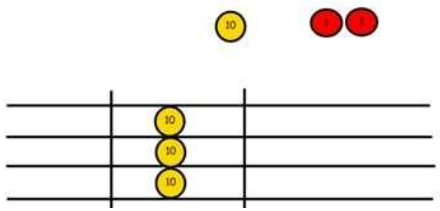
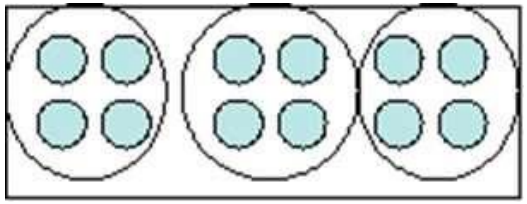
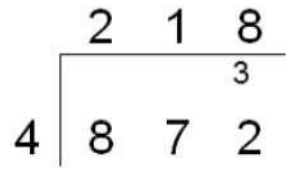
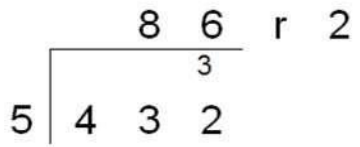
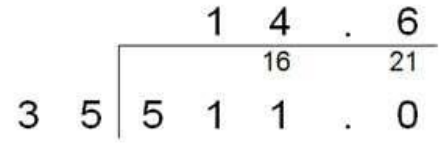
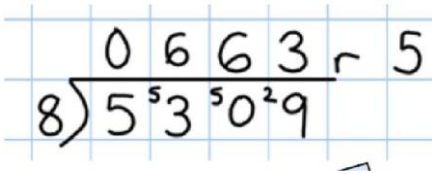
Year 4- 6 Division			
Objective/ Strategy	Concrete	Pictorial	Abstract
Divide at least 3-digit numbers by 1 digit. Short Division	 Use place value counters to divide using the bus stop method alongside 42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over. 	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder.  Move onto divisions with a remainder.  Finally move into decimal places to divide the total accurately.  

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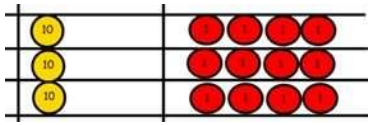
	We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.		

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